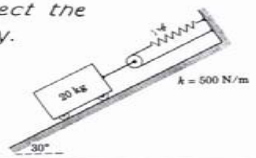
EXEMPLOS

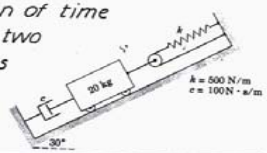
ART. 8/2 FREE VIBRATION OF PARTICLES (UNDAMPED) NO.

The 20-kg body is displaced a distance of 200 mm down the incline from its equilibrium position and released from rest. Determine the natural frequency f_n of its subsequent oscillation and its maximum velocity and acceleration. Neglect the mass of the pulley.



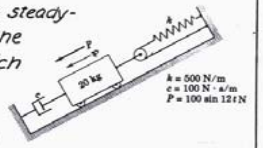
ART. 8/2 FREE VIBRATION OF PARTICLES (DAMPED) NO. 2

The 20-kg body is displaced a distance of 200 mm down the incline from its equilibrium position and released from rest. Express the displacement x from the equilibrium position as a function of time and find the ratio of two successive amplitudes of the damped oscillation.



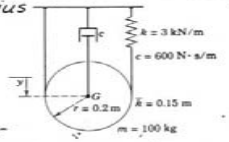
ART. 8/3 FORCED VIBRATION OF PARTICLES

The 20-kg body is subjected to a periodic force P = 100 sin 12t newtons where t is in seconds. With spring stiffness k and viscous damping constant c as shown, determine the amplitude X of the steady-state vibration and the phase angle ϕ by which X lags P. Also find the amount ΔX by which the undamped amplitude is reduced.



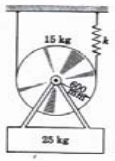
ART. 8/4 VIBRATION OF RIGID BODIES

The 100-kg wheel with radius of gyration $\bar{k}$ is released from rest with G displaced 0.2 m below the equilibrium position. With spring stiffness k and damping constant c shown, derive the equation of motion and express y as a function of time. Find the ratio of two successive amplitudes.



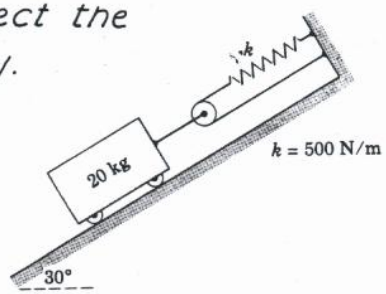
ART. 8/5 ENERGY METHODS

Find the natural frequency f_n of vertical oscillation of the 25-kg mass after release from a displaced position. The radius of gyration of the 15-kg pulley is $\bar{k}$ = 450 mm, and the spring stiffness is k = 540 N/m.



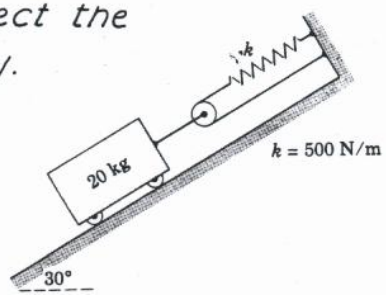
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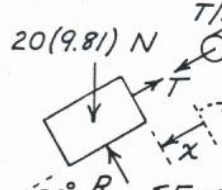


ART. 8/2 FREE VIBRATION OF PARTICLES (UNDAMPED) NO.

The 20-kg body is displaced a distance of 200 mm down the incline from its equilibrium position and released from rest. Determine the natural frequency f_n of its subsequent oscillation and its maximum velocity and acceleration. Neglect the mass of the pulley.



In equilibrium position, $T = T_0 = 20(9.81)\sin 30° = 98.1$ N



In general position, increase in spring tension is $\Delta(\frac{T}{2}) = k(2x)$ ⇒

$T = T_0 + \Delta T = 98.1 + 4kx = 98.1 + 2000x$ N

$\Sigma F_x = m\ddot{x}$: $20(9.81)\sin 30° - (98.1 + 2000x) = 20\ddot{x}$

or $98.1 - 98.1 - 2000x = 20\ddot{x}$, $\ddot{x} + 100x = 0$

so $\omega_n = \sqrt{100} = 10$ rad/s, $f_n = \frac{\omega_n}{2\pi} = \frac{10}{2\pi} = \boxed{1.59 \text{ Hz}}$

Sol. (Eq. 8/6) is $x = x_0 \cos \omega_n t + (\dot{x}_0/\omega_n)\sin \omega_n t$

where $x_0 = 0.200$ m, $\dot{x}_0 = 0$. Thus $x = 0.2 \cos 10t$

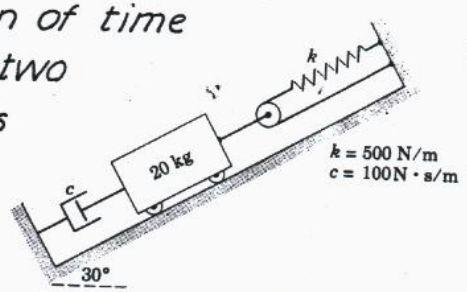
⇒ $\dot{x} = -0.2(10)\sin 10t$, $\boxed{|\dot{x}_{max}| = 2 \text{ m/s}}$

$\ddot{x} = -0.2(10)^2 \cos 10t$, $\boxed{|\ddot{x}_{max}| = 20 \text{ m/s}^2}$

JLM

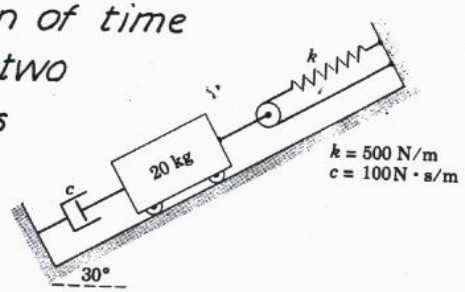
ART. 8/2 FREE VIBRATION OF PARTICLES (DAMPED) NO. 2

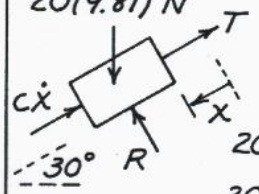
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The 20-kg body is displaced a distance of 200 mm down the incline from its equilibrium position and released from rest. Express the displacement x from the equilibrium position as a function of time and find the ratio of two successive amplitudes of the damped oscillation.





$T = T_0 + Kx = 98.1 + 2000x$ N (see Sol. No. 1 for equiv. spr. const. $K = 4k = 2000$ N/m)

$\Sigma F_x = m\ddot{x}$ gives

$20(9.81)\sin 30° - (98.1 + 2000x) - 100\dot{x} = 20\ddot{x}$

$20\ddot{x} + 100\dot{x} + 2000x = 0, \quad \ddot{x} + 5\dot{x} + 100x = 0$

Solution (Eq. 8/12) is $x = Ce^{-\zeta\omega_n t}\sin(\omega_d t + \psi)$

$\omega_n = \sqrt{K/m} = \sqrt{2000/20} = 10$ rad/s, $\zeta = \frac{c}{2m\omega_n} = \frac{100}{2(20)(10)} = 0.25$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 10\sqrt{1-(0.25)^2} = 9.68$ rad/s

$x = Ce^{-2.5t}\sin(9.68t + \psi)$; $x = 0.2$ m & $\dot{x} = 0$ for $t = 0$ give

$0.2 = C\sin\psi$ & $0 = -2.5C\sin\psi + 9.68C\cos\psi$. Solve & get

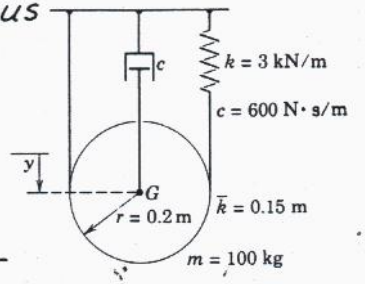
$C = 0.207$ m, $\psi = 1.318$ rad so $\boxed{x = 0.207e^{-2.5t}\sin(9.68t + 1.318)}$

$\delta = \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \frac{2\pi(0.25)}{\sqrt{1-(0.25)^2}} = 1.622, \quad \boxed{x_1/x_2 = 5.06}$

JLM

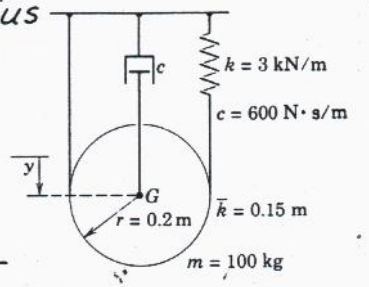
ART. 8/4 VIBRATION OF RIGID BODIES

The 100-kg wheel with radius of gyration $\bar{k}$ is released from rest with G displaced 0.2 m below the equilibrium position. With spring stiffness k and damping constant c shown, derive the equation of motion and express y as a function of time. Find the ratio of two successive amplitudes.



ART. 8/4 VIBRATION OF RIGID BODIES

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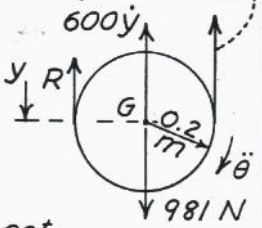
Equil. $T_0 = 981/2$ N, $T = 490.5 + 6000y$ $\quad T = T_0 + 3000(2y)$

$\Sigma M_G = \bar{I}\ddot{\theta}$

$0.2R - 0.2(490.5 + 6000y) = 100(0.15)^2\ddot{\theta}$

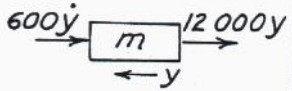
$\Sigma F_y = m\ddot{y}$

$981 - [R + 600\dot{y} + 490.5 + 6000y] = 100\ddot{y}$



Eliminate R, substitute $\ddot{\theta} = \ddot{y}/0.2$ & get

$$\boxed{156.25\ddot{y} + 600\dot{y} + 12\,000y = 0}$$



Motion is equivalent to that of a particle of mass $m = 156.25$ kg, $k = 12\,000$ N/m, $c = 600$ N·s/m

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{12\,000}{156.25}} = 8.76 \frac{\text{rad}}{\text{s}}$, $\vartheta = \frac{c}{2m\omega_n} = \frac{600}{2(156.25)(8.76)} = 0.219$

$\omega_d = \omega_n\sqrt{1-\vartheta^2} = 8.76\sqrt{1-(0.219)^2} = 8.55$ rad/s

From Eq. 8/12, $y = Ce^{-\vartheta\omega_n t}\sin(\omega_d t + \psi)$

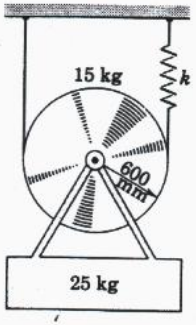
With $y = 0.2$ m, $\dot{y} = 0$ for $t = 0$, $\boxed{y = 0.205e^{-1.92t}\sin(8.55t + 1.350)}$

$\delta = \ln\left(\frac{y_1}{y_2}\right) = \frac{2\pi\vartheta}{\sqrt{1-\vartheta^2}} = 2\pi(0.219)/\sqrt{1-(0.219)^2} = 1.411$, $\boxed{y_1/y_2 = 4.10}$

JLM

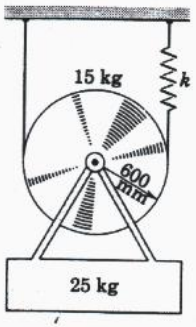
ART. 8/5 ENERGY METHODS

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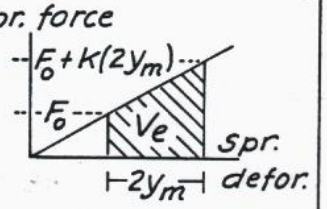


Let y = downward displacement from equil. position

Take $V_e = 0$ & $V_g = 0$ when $y = 0$

At equil. $F_0 = \frac{1}{2}(40)(9.81) = 196.2$ N

For max. displacement $y = y_m$



$$V_g = -40(9.81)y_m = -392.4y_m$$

$$V_e = 196.2(2y_m) + \tfrac{1}{2}(540)(2y_m)^2 = 392.4y_m + 1080y_m^2$$

$$V_{max} = V_g + V_e = 1080y_m^2$$

For $y = 0$, $T = T_{max} = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$

$$= \tfrac{1}{2}(40)\bar{v}^2 + \tfrac{1}{2}(15)(0.450)^2(\bar{v}/0.6)^2$$

$$= 24.2\bar{v}^2 \text{ where } \bar{v} = v_{max}$$

For linear response $y = y_m \sin \omega_n t$

$v_{max} = \dot{y}_{max} = y_m \omega_n$ so $T_{max} = 24.2y_m^2\omega_n^2$

But $T_{max} = V_{max}$ so $24.2y_m^2\omega_n^2 = 1080y_m^2$

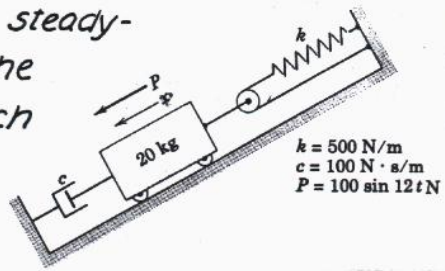
$\omega_n^2 = 1080/24.2$, $\omega_n = 6.68$ rad/s

$f_n = \omega_n/2\pi = 6.68/2\pi = \boxed{1.063 \text{ Hz}}$

JLM

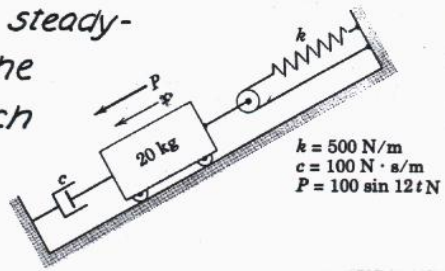
ART. 8/3 FORCED VIBRATION OF PARTICLES

The 20-kg body is subjected to a periodic force $P = 100 \sin 12t$ newtons where t is in seconds. With spring stiffness k and viscous damping constant c as shown, determine the amplitude X of the steady-state vibration and the phase angle ϕ by which X lags P. Also find the amount ΔX by which the undamped amplitude is reduced.

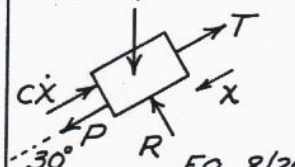


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$20(9.81)$ N



$T = T_0 + Kx = 98.1 + 2000x$ N (see No. 1 of 8/2 for equiv. spr. const. $K = 4k = 2000$ N/m)

$P = P_0 \sin \omega t = 100 \sin 12t$ N

Eq. 8/20, $X = \dfrac{P_0/K}{\left\{[1-(\omega/\omega_n)^2]^2 + [2\beta\omega/\omega_n]^2\right\}^{1/2}}$

$\omega_n = \sqrt{\dfrac{K}{m}} = \sqrt{\dfrac{2000}{20}} = 10 \dfrac{\text{rad}}{\text{s}}$, $\beta = \dfrac{c}{2m\omega_n} = \dfrac{100}{2(20)(10)} = 0.25$

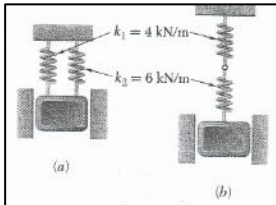
$$X = \frac{100/2000}{\left\{[1-(12/10)^2]^2 + [2(0.25)(12/10)]^2\right\}^{1/2}} = \boxed{0.0672 \text{ m}}$$

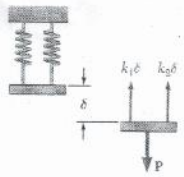
Eq. 8/21, $\phi = \tan^{-1}\left[\dfrac{2\beta\omega/\omega_n}{1-(\omega/\omega_n)^2}\right] = \tan^{-1}\left[\dfrac{2(0.25)(12/10)}{1-(12/10)^2}\right] = \boxed{0.701\pi \text{ rad}}$

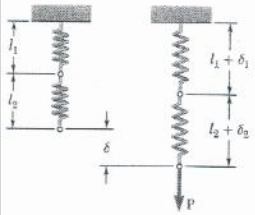
For $c = 0$, $X = \left|\dfrac{100/2000}{1-(12/10)^2}\right| = 0.1136$ m, $\Delta X = 0.1136 - 0.0672$

$= \boxed{0.0464 \text{ m}}$

JLM







Um bloco de 50 kg se move entre guias verticais, como mostra a figura. O bloco é puxado até 40 mm abaixo de sua posição de equilíbrio e liberado. Para cada combinação de molas, determine o período da vibração, a velocidade máxima do bloco e a aceleração máxima desse bloco.

SOLUÇÃO

a. Molas presas em paralelo. Inicialmente determinamos a constante k de uma mola única equivalente às duas molas, *encontrando a intensidade da força* **P** necessária para produzir uma dada deflexão δ. Uma vez que para uma deflexão δ as intensidades das forças exercidas pelas molas são, respectivamente, $k_1\delta$ e $k_2\delta$, temos

$$P = k_1\delta + k_2\delta = (k_1 + k_2)\delta$$

A constante k da mola única equivalente é

$$k = \frac{P}{\delta} = k_1 + k_2 = 4\text{ kN/m} + 6\text{ kN/m} = 10\text{ kN/m} = 10^4\text{ N/m}$$

Período de vibração: Como $m = 50$ kg, a Eq. (19.4) resulta em

$$\omega_n^2 = \frac{k}{m} = \frac{10^4\text{ N/m}}{50\text{ kg}} \qquad \omega_n = 14{,}14\text{ rad/s}$$

$$\tau_n = 2\pi/\omega_n \qquad \boldsymbol{\tau_n = 0{,}444\text{ s}} \blacktriangleleft$$

Velocidade máxima: $v_m = x_m\omega_n = (0{,}040\text{ m})(14{,}14\text{ rad/s})$

$$v_m = 0{,}566\text{ m/s} \qquad \mathbf{v_m = 0{,}566\text{ m/s}\uparrow\downarrow} \blacktriangleleft$$

Aceleração máxima: $a_m = x_m\omega_n^2 = (0{,}040\text{ m})(14{,}14\text{ rad/s})^2$

$$a_m = 8{,}00\text{ m/s}^2 \qquad \mathbf{a_m = 8{,}00\text{ m/s}^2\uparrow\downarrow} \blacktriangleleft$$

b. Molas presas em série. Determinamos inicialmente a constante k da mola única equivalente às duas molas, *encontrando a elongação total* δ dessas molas sob a ação de uma carga estática dada **P**. Visando facilitar os cálculos, uma carga estática de intensidade $P = 12$ kN é utilizada.

$$\delta = \delta_1 + \delta_2 = \frac{P}{k_1} + \frac{P}{k_2} = \frac{12\text{ kN}}{4\text{ kN/m}} + \frac{12\text{ kN}}{6\text{ kN/m}} = 5\text{ m}$$

$$k = \frac{P}{\delta} = \frac{12\text{ kN}}{5\text{ m}} = 2{,}4\text{ kN/m} = 2.400\text{ N/m}$$

Período de vibração: $\omega_n^2 = \dfrac{k}{m} = \dfrac{2.400\text{ N/m}}{50\text{ kg}}$ $\omega_n = 6{,}39$ rad/s

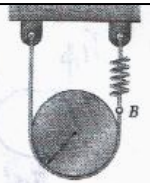
$$\tau_n = \frac{2\pi}{\omega_n} \qquad \boldsymbol{\tau_n = 0{,}907\text{ s}} \blacktriangleleft$$

Velocidade máxima: $v_m = x_m\omega_n = (0{,}040\text{ m})(6{,}93\text{ rad/s})$

$$v_m = 0{,}277\text{ m/s} \qquad \mathbf{v_m = 0{,}277\text{ m/s}\uparrow\downarrow} \blacktriangleleft$$

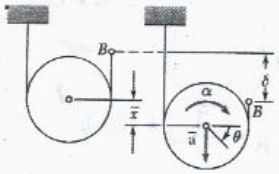
Aceleração máxima: $a_m = x_m\omega_n^2 = (0{,}040\text{ m})(6{,}93\text{ rad/s})^2$

$$a_m = 1{,}920\text{ m/s}^2 \qquad \mathbf{a_m = 1.920\text{ m/s}^2\uparrow\downarrow} \blacktriangleleft$$



Um cilindro de peso W e raio r está suspenso por um laço de corda, conforme mostrado. Uma extremidade da corda está presa diretamente a um suporte rígido, enquanto a outra extremidade está presa a uma mola de constante k. Determine o período e a freqüência natural das vibrações do cilindro.

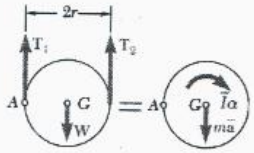
SOLUÇÃO



Cinemática do movimento. Expressamos o deslocamento liñear e a aceleração do cilindro em função do deslocamento angular θ. Escolhendo o sentido horário como positivo e medindo os deslocamentos a partir da posição de equilíbrio, escrevemos

$$\bar{x} = r\theta \qquad \delta = 2\bar{x} = 2r\theta$$
$$\alpha = \ddot{\theta}\downarrow \qquad \bar{a} = r\alpha = r\ddot{\theta} \qquad \bar{\mathbf{a}} = r\ddot{\theta}\downarrow \tag{1}$$

Equações de movimento. O sistema de forças externas que atuam sobre o cilindro consiste no peso $\mathbf{W}$ e nas forças $\mathbf{T}_1$ e $\mathbf{T}_2$ exercidas pela corda. Expressamos que este sistema é equivalente ao sistema de forças efetivas representado pelo vetor $m\bar{\mathbf{a}}$ ligado a G e o binário $\bar{I}\alpha$.



$$+\downarrow \Sigma M_A = \Sigma(M_A)_{\text{efet.}}: \qquad Wr - T_2(2r) = m\bar{a}r + \bar{I}\alpha \tag{2}$$

Quando o cilindro está em sua posição de equilíbrio, a tração na corda é $T_0 = \frac{1}{2}W$. Verificamos que, para um deslocamento angular θ, a intensidade de $\mathbf{T}_2$ é

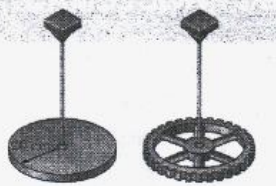
$$T_2 = T_0 + k\delta = \tfrac{1}{2}W + k\delta = \tfrac{1}{2}W + k(2r\theta) \tag{3}$$

Substituindo (1) e (3) em (2) e lembrando que $\bar{I} = \frac{1}{2}mr^2$, escrevemos

$$Wr - (\tfrac{1}{2}W + 2kr\theta)(2r) = m(r\ddot{\theta})r + \tfrac{1}{2}mr^2\ddot{\theta}$$
$$\ddot{\theta} + \frac{8}{3}\frac{k}{m}\theta = 0$$

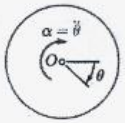
Vemos que o movimento é harmônico simples, e temos

$$\omega_n^2 = \frac{8}{3}\frac{k}{m} \qquad \omega_n = \sqrt{\frac{8}{3}\frac{k}{m}}$$
$$\tau_n = \frac{2\pi}{\omega_n} \qquad \tau_n = 2\pi\sqrt{\frac{3}{8}\frac{m}{k}} \blacktriangleleft$$
$$f_n = \frac{\omega_n}{2\pi} \qquad f_n = \frac{1}{2\pi}\sqrt{\frac{8}{3}\frac{k}{m}} \blacktriangleleft$$



Um disco circular pesando 90 N e de raio de 20 cm, está suspenso por um arame como ilustrado na figura. O disco é girado (torcendo, portanto, o arame) e em seguida liberado; o período da vibração torcional é visto como sendo de 1,13 s. A seguir, uma engrenagem é então suspensa pelo mesmo arame e o período de vibração torcional é observado como de 1,93 s. Considerando que o momento do binário exercido pelo arame é proporcional ao ângulo de torção, determine (*a*) a constante de mola torcional do arame, (*b*) o momento de inércia centroidal da engrenagem, (*c*) a velocidade angular máxima alcançada pela engrenagem quando ela é girada em 90° e liberada.

SOLUÇÃO



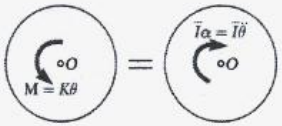
***a.* Vibração do disco.** Representando por θ o deslocamento angular do disco, expressamos que a intensidade do binário exercido pelo arame é $M = K\theta$, onde K é a constante da mola torcional do arame. Uma vez que este binário deve ser equivalente ao binário $\bar{I}\alpha$ que representa as forças efetivas do disco, escrevemos

$$+\circlearrowleft \Sigma M_O = \Sigma (M_O)_{\text{efet}}: \qquad +K\theta = -\bar{I}\ddot{\theta}$$

$$\ddot{\theta} + \frac{K}{\bar{I}}\theta = 0$$

O movimento é, portanto, harmônico simples, e temos

$$\omega_n^2 = \frac{K}{\bar{I}} \qquad \tau_n = \frac{2\pi}{\omega_n} \qquad \tau_n = 2\pi\sqrt{\frac{\bar{I}}{K}} \tag{1}$$



Para o disco, temos

$$\tau_n = 1{,}13 \text{ s} \qquad \bar{I} = \tfrac{1}{2}mr^2 = \frac{1}{2}\left(\frac{90 \text{ N}}{9{,}81 \text{ m/s}^2}\right)(0{,}2 \text{ m})^2 = 0{,}184 \text{ kg} \cdot \text{m}^2$$

Substituindo em (1), obtemos

$$1{,}13 = 2\pi\sqrt{\frac{0{,}184}{K}} \qquad \mathbf{K = 5{,}7 \text{ N} \cdot \text{m/rad}} \blacktriangleleft$$

***b.* Vibração da engrenagem.** Como o período de vibração da engrenagem é de 1,93 s e $K = 5{,}7$ N · m/rad, a Eq (1) fornece

$$1{,}93 = 2\pi\sqrt{\frac{\bar{I}}{5{,}7}} \qquad \mathbf{\bar{I}_{engr.} = 0{,}54 \text{ kg} \cdot \text{m}^2} \blacktriangleleft$$

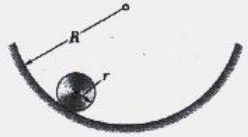
***c.* Velocidade angular máxima da engrenagem.** Como o movimento é harmônico simples, temos

$$\theta = \theta_m \operatorname{sen} \omega_n t \qquad \omega = \theta_m \omega_n \cos \omega_n t \qquad \omega_m = \theta_m \omega_n$$

Recordando que $\theta_m = 90° = 1{,}571$ rad e $\tau = 1{,}93$ s, escrevemos

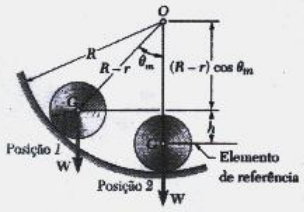
$$\omega_m = \theta_m \omega_n = \theta_m \left(\frac{2\pi}{\tau}\right) = (1{,}571 \text{ rad})\left(\frac{2\pi}{1{,}93 \text{ s}}\right)$$

$$\mathbf{\omega_m = 5{,}11 \text{ rad/s}} \blacktriangleleft$$



Determine o período de pequenas oscilações de um cilindro de raio r que rola sem deslizar no interior de uma superfície curva de raio R.

SOLUÇÃO



Representamos por θ o ângulo que a reta OG forma com a vertical. Como o cilindro rola sem deslizar, podemos aplicar o princípio de conservação da energia entre a posição *1*, onde $\theta = \theta_m$, e a posição 2, onde $\theta = 0$.

Posição *1*

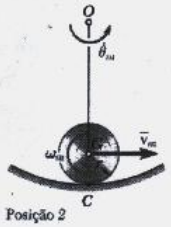
Energia cinética. Como a velocidade do cilindro é zero, $T_1 = 0$.

Energia potencial. Escolhendo uma referência como mostrado na figura e representando por W o peso do cilindro, temos

$$V_1 = Wh = W(R-r)(1-\cos\theta)$$

Verificando que para pequenas oscilações $(1-\cos\theta) = 2\,\mathrm{sen}^2(\theta/2) \approx \theta^2/2$, temos

$$V_1 = W(R-r)\frac{\theta_m^2}{2}$$



Posição 2. Representando por $\dot{\theta}_m$ a velocidade angular da reta OG quando o cilindro passa pela posição 2, e observando que o ponto C é o centro instantâneo de rotação do cilindro, escrevemos

$$\bar{v}_m = (R-r)\dot{\theta}_m \qquad \omega_m = \frac{\bar{v}_m}{\bar{r}} = \frac{R-r}{r}\dot{\theta}_m$$

Energia cinética

$$T_2 = \tfrac{1}{2}m\bar{v}_m^2 + \tfrac{1}{2}\bar{I}\omega_m^2$$

$$= \tfrac{1}{2}m(R-r)^2\dot{\theta}_m^2 + \tfrac{1}{2}\left(\tfrac{1}{2}mr^2\right)\left(\frac{R-r}{r}\right)^2\dot{\theta}_m^2$$

$$= \tfrac{3}{4}m(R-r)^2\dot{\theta}_m^2$$

Energia potencial

$$V_2 = 0$$

Conservação de energia

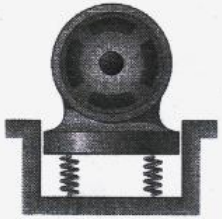
$$T_1 + V_1 = T_2 + V_2$$

$$0 + W(R-r)\frac{\theta_m^2}{2} = \tfrac{3}{4}m(R-r)^2\dot{\theta}_m^2 + 0$$

Como $\dot{\theta}_m = \omega_n\theta_m$ e $W = mg$, escrevemos

$$mg(R-r)\frac{\theta_m^2}{2} = \tfrac{3}{4}m(R-r)^2(\omega_n\theta_m)^2 \qquad \omega_n^2 = \frac{2}{3}\frac{g}{R-r}$$

$$\tau_n = \frac{2\pi}{\omega_n} \qquad \tau_n = 2\pi\sqrt{\frac{3}{2}\frac{R-r}{g}} \quad \blacktriangleleft$$



PROBLEMA RESOLVIDO 19.5

Um motor que pesa 1.575 N é suportado por quatro molas, cada uma tendo uma constante de 1.350 N/cm. O desbalanceamento do rotor é equivalente a um peso de 0,28 N localizado a 15 cm do eixo de rotação. Sabendo que o motor é restringido a mover-se verticalmente, determine (*a*) a rotação em rpm na qual ocorrerá ressonância, (*b*) a amplitude da vibração do motor a uma rotação de 1.200 rpm.

SOLUÇÃO

***a.* Rotação de ressonância.** A rotação de ressonância é igual à freqüência natural circular ω_n (em rpm) da vibração livre do motor. A massa do motor e a constante equivalente das molas de sustentação são

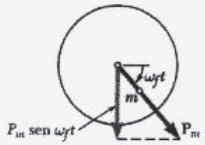
$$m = \frac{1.575 \text{ N}}{9,81 \text{ m/s}^2} = 160,55 \text{ kg}$$

$$k = (1.350 \text{ N/cm}) = 5.400 \text{ N/cm} = 540.000 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{540.000}{160,55}} = 58 \text{ rad/s} = 554 \text{ rpm}$$

Rotação de ressonância = 554 rpm ◀

***b.* Amplitude de vibração a 1.200 rpm.** A velocidade angular do motor e a massa equivalente ao peso de 0,28 N são



$$\omega = 1.200 \text{ rpm} = 125,7 \text{ rad/s}$$

$$m = \frac{P}{g} = \frac{0,28 \text{ N}}{9,81 \text{ m/s}^2} = 0,0285 \text{ kg}$$

A intensidade da força centrífuga causada pelo desbalanceamento do rotor é

$$P_m = ma_n = mr\omega^2 = (0,0285 \text{ kg})(0,15 \text{ m})(125,7 \text{ rad/s})^2 = 67,55 \text{ N}$$

A deflexão estática que seria causada por uma carga constante P_m é

$$\frac{P_m}{k} = \frac{67,55 \text{ N}}{5.400 \text{ N/cm}} = 0,0125 \text{ cm}$$

A freqüência forçada circular ω_f do movimento é a velocidade angular do motor,

$$\omega_f = \omega = 125,7 \text{ rad/s}$$

Substituindo os valores de P_m/k, ω_f e ω_n na Eq. (19.33), obtemos

$$x_m = \frac{P_m/k}{1-(\omega_f/\omega_n)^2} = \frac{0,0125 \text{ cm}}{1-(125,7/58)^2} = -0,00338 \text{ cm}$$

$x_m = 0,00338$ cm (defasado) ◀

Nota. Como $\omega_f > \omega_n$, a vibração está defasada 180° em relação à força centrífuga causada pelo desbalanceamento do rotor. Por exemplo, quando a massa desbalanceada está diretamente abaixo do eixo de rotação, a posição do motor é $x_m = 0,00338$ cm acima da posição de equilíbrio.