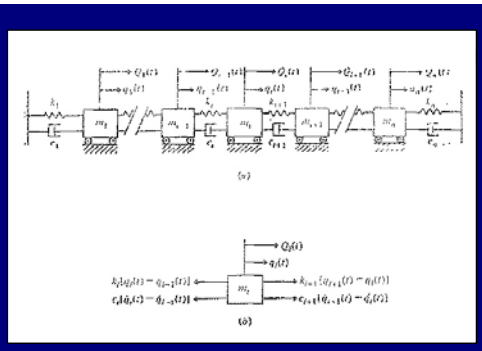
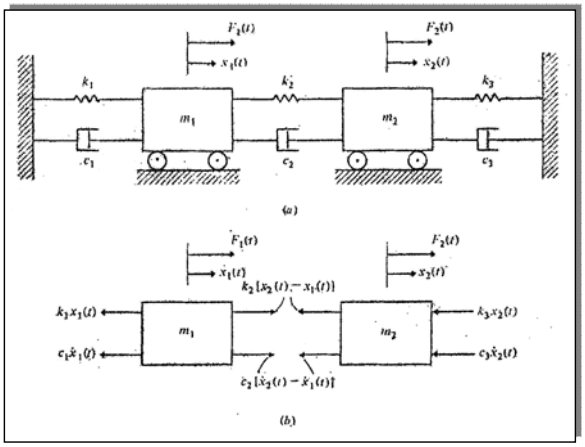


Cap. VI – Sistemas com mais de 1 Grau de Liberdade

Sistemas de 2 graus de liberdade



$$m_1\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 - c_2\dot{x}_2 = F_1$$
$$m_2\ddot{x}_2 + (c_2 + c_3)\dot{x}_2 + (k_2 + k_3)x_2 - k_2x_1 - c_2\dot{x}_1 = F_2$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} (c_1 + c_2) & -c_2 \\ -c_2 & (c_2 + c_3) \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} (k_1 + k_2) & -k_2 \\ -k_2 & (k_2 + k_3) \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

Sistemas de 2 graus de liberdade

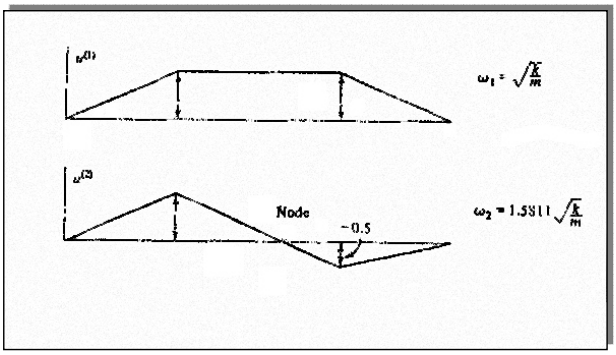
$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

Para $\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$ (Vibração livre):

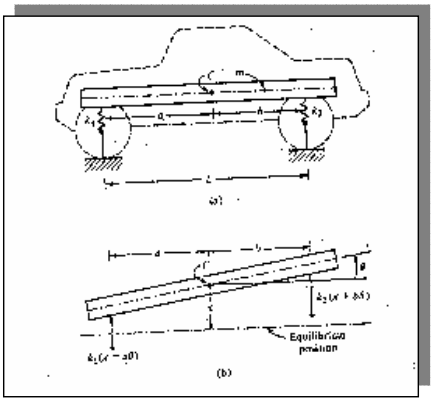
$$\begin{matrix} \omega_{n_1}^2 \\ \omega_{n_2}^2 \end{matrix} = \frac{1}{2}\frac{m_1 k_{22} + m_2 k_{11}}{m_1 m_2} \pm \frac{1}{2}\sqrt{\left(\frac{m_1 k_{22} + m_2 k_{11}}{m_1 m_2}\right)^2 - 4\frac{k_{11}k_{22} - k_{12}^2}{m_1 m_2}}$$

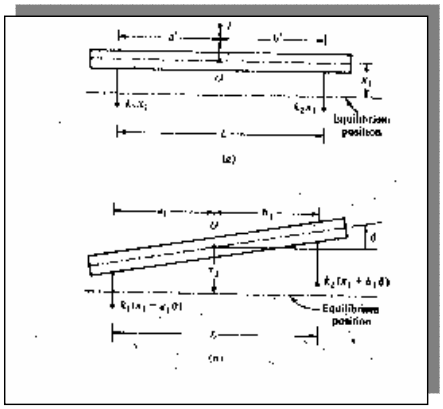
$$\left\{u^{(1)}\right\} = \begin{Bmatrix} u_1^{(1)} \\ u_2^{(1)} \end{Bmatrix} \rightarrow \omega_{n_1}$$

$$\left\{u^{(2)}\right\} = \begin{Bmatrix} u_1^{(2)} \\ u_2^{(2)} \end{Bmatrix} \rightarrow \omega_{n_2}$$

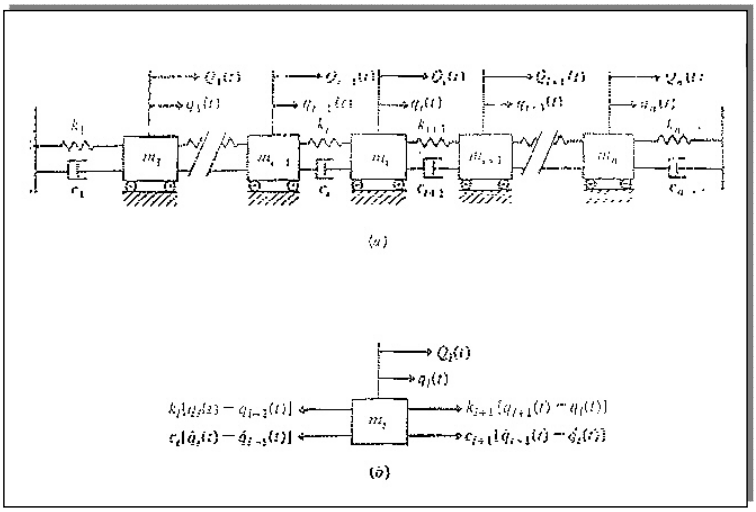


Sistemas de 2 graus de liberdade



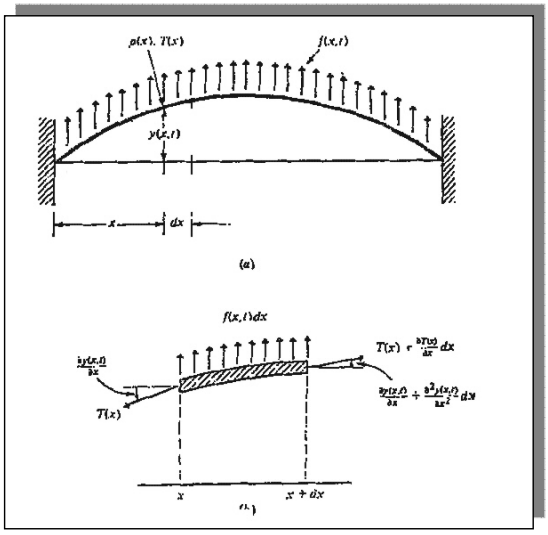


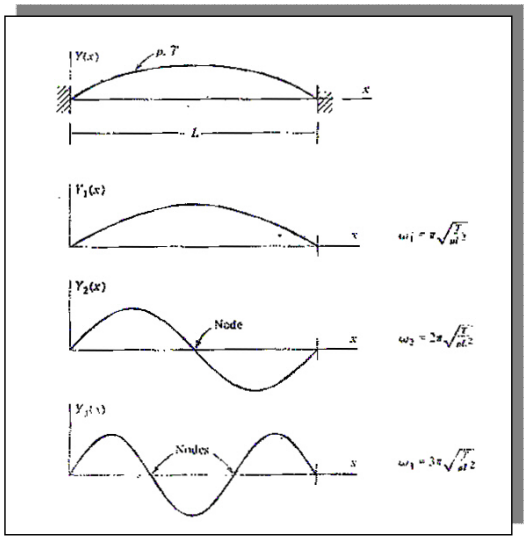
Sistemas de N graus de liberdade



$$[m]\{\ddot{q}(t)\}+[c]\{\dot{q}(t)\}+[k]\{q(t)\}=\{Q(t)\}$$

Sistemas Contínuos
Ex.: Corda





$$\frac{\partial}{\partial x}\left[T(x)\frac{\partial y(x,t)}{\partial x}\right] + f(x,t) = \rho(x)\frac{\partial^2 y(x,t)}{\partial t^2}$$