Pares de Transformadas de Laplace		
	$f(t)$	$F(s)$
1	*Impulso Unitário* $\delta(t)$	1
2	*Degrau Unitário* $1(t)$	$\frac{1}{s}$
3	*Rampa Unitária* t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n=1,2,3,\cdots)$	$\frac{1}{s^n}$
5	$t^n \quad (n=1,2,3,\cdots)$	$\frac{n!}{s^{n+1}}$
6	$e^{\mp at}$	$\frac{1}{s \pm a}$
7	$t e^{\mp at}$	$\frac{1}{(s \pm a)^2}$
8	$\frac{1}{(n-1)!} t^{n-1} e^{-at} \quad (n=1,2,3,\cdots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \quad (n=1,2,3,\cdots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$
11	$\frac{1}{b-a}(e^{-at}-e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
12	$\frac{1}{b-a}(b e^{-bt}-a e^{-at})$	$\frac{s}{(s+a)(s+b)}$
13	$\frac{1}{ab}\left[1+\frac{1}{a-b}(b e^{-at}-a e^{-bt})\right]$	$\frac{1}{s(s+a)(s+b)}$
14	$\frac{1}{a^2}(1-e^{-at}-a t e^{-at})$	$\frac{1}{s(s+a)^2}$
15	$\frac{1}{a^2}(at-1+e^{-at})$	$\frac{1}{s^2(s+a)}$
16	$\frac{1}{a^2}\left[t-\frac{2}{a}+\left(t+\frac{2}{a}\right)e^{-at}\right]$	$\frac{1}{s^2(s+a)^2}$
17	$(1-at)e^{-at}$	$\frac{s}{(s+a)^2}$
18	$\frac{1}{a^2}[b-be^{-at}+(a-b)a t e^{-at}]$	$\frac{s+b}{s(s+a)^2}$

19	$\frac{e^{-at}}{(b-a)(c-a)}+\frac{e^{-bt}}{(c-b)(a-b)}+\frac{e^{-ct}}{(a-c)(b-c)}$	$\frac{1}{(s+a)(s+b)(s+c)}$
20	$\frac{(d-a)e^{-at}}{(b-a)(c-a)}+\frac{(d-b)e^{-bt}}{(c-b)(a-b)}+\frac{(d-c)e^{-ct}}{(a-c)(b-c)}$	$\frac{s+d}{(s+a)(s+b)(s+c)}$
21	$sen\ \omega t$	$\frac{\omega}{s^2+\omega^2}$
22	$cos\ \omega t$	$\frac{s}{s^2+\omega^2}$
23	$e^{-at} sen\ \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
24	$e^{-at} cos\ \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
25	$\omega \sqrt{a^2+\omega^2}\ sen\ (\omega t+\varphi)$ $\varphi = tag^{-1}(\omega/a)$	$\frac{\omega^2(s+a)}{s^2+\omega^2}$
26	$\frac{\omega}{a^2+\omega^2} e^{-at} + \frac{1}{\sqrt{a^2+\omega^2}}\ sen\ (\omega t - \varphi)$ $\varphi = tag^{-1}(\omega/a)$	$\frac{\omega}{(s+a)(s^2+\omega^2)}$
27	$\frac{1}{\omega}\sqrt{(z-a)^2+\omega^2}\ e^{-at}\ sen\ (\omega t + \varphi)$ $\varphi = tag^{-1}[\omega/(z-a)]$	$\frac{s+z}{(s+a)^2+\omega^2}$
28	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\ sen\ \omega_n\sqrt{1-\zeta^2}\,t\ , \quad \zeta < 1$	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
29	$-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\ sen\left(\omega_n\sqrt{1-\zeta^2}\ t-\beta\right)$ $\beta = tag^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta} = cos^{-1}\zeta \ ; \ \zeta < 1$	$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$
30	$1-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\ sen\left(\omega_n\sqrt{1-\zeta^2}\ t+\beta\right)$ $\beta = tag^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta} = cos^{-1}\zeta \ ; \ \zeta < 1$	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$
31	$t-\frac{2\zeta}{\omega_n}+\frac{1}{\omega_n\sqrt{1-\zeta^2}}\ e^{-\zeta\omega_n t}\ sen\ (\omega_n\sqrt{1-\zeta^2}\ t+\varphi)$ $\varphi = cos^{-1}(2\zeta^2-1) \ ; \ \zeta < 1$	$\frac{\omega_n^2}{s^2(s^2+2\zeta\omega_n s+\omega_n^2)}$

32	$\frac{1}{a^2+\omega^2}+\frac{1}{\omega\sqrt{a^2+\omega^2}}e^{-at}\,sen\left(\omega t-\varphi\right)$ $\varphi=tag^{-1}\,\frac{\omega}{-a}$	$\frac{1}{s[(s+a)^2+\omega^2]}$
33	$\frac{z}{a^2+\omega^2}+\frac{1}{\omega}\left[\frac{(z-a)^2+\omega^2}{a^2+\omega^2}\right]^{1/2}e^{-at}\,sen\left(\omega t+\varphi\right)$ $\varphi=tag^{-1}\,\frac{\omega}{z-a}-tag^{-1}\,\frac{\omega}{-a}$	$\frac{s+z}{s[(s+a)^2+\omega^2]}$
34	$1-\cos\omega t$	$\frac{\omega^2}{s(s^2+\omega^2)}$
35	$\omega t-sen\,\omega t$	$\frac{\omega^3}{s^2(s^2+\omega^2)}$
36	$sen\,\omega t-\omega t\cos\omega t$	$\frac{2\omega^3}{(s^2+\omega^2)^2}$
37	$\frac{1}{2\omega}t\,sen\,\omega t$	$\frac{s}{(s^2+\omega^2)^2}$
38	$t\cos\omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
39	$\frac{1}{\omega_2^2-\omega_1^2}(\cos\omega_1 t-\cos\omega_2 t)\qquad(\omega_1^2\neq\omega_2^2)$	$\frac{s}{(s^2+\omega_1^2)(s^2+\omega_2^2)}$
40	$\frac{1}{2\omega}(sen\,\omega t+\omega t\cos\omega t)$	$\frac{s^2}{(s^2+\omega^2)^2}$

Convolução no tempo ⇔ Multiplicação Complexa:

$$F_1(s).F_2(s)=L\left[\int_0^t f_1(\tau)\;f_2(t-\tau)d\tau\right]=L\left[\int_0^t f_1(t-\tau)\,f_2(\tau)d\tau\right]=L\left[f_1(t)*f_2(t)\right]$$

com $f_1(t) = f_2(t) = 0$ *para* $t < 0$

Teorema do Valor Inicial: $f(0^+)=lim_{t\to 0}\;f(t)=lim_{s\to\infty}\;s\,F(s)$

Teorema do Valor Final: $f(\infty)=lim_{t\to\infty}\;f(t)=lim_{s\to 0}\;s\,F(s)$

Obs.: Só se aplica (sse) $lim_{t\to\infty}\;f(t)$ *existe (todos os polos no SPE, exceto na origem)*

Teorema de Euler: $e^{\pm j\theta}=\cos\theta\pm j\,sen\,\theta$

Obs.: $e^{\pm st}=e^{(\sigma t\pm j\omega t)}=e^{\sigma t}\,e^{\pm j\omega t}=e^{\sigma t}\,(\cos\omega t\pm j\,sen\,\omega t)$

Propriedades das Transformadas de Laplace	
1	$L[A\ f(t)] = A\,F(s)$
2	$L[\ f_1(t) \pm f_2(t)\ ] = F_1(s) \pm F_2(s)$
3	$L_{\pm}\left[\dfrac{d\ f(t)}{dt}\right] = s\,F(s) - f(0\pm)$
4	$L_{\pm}\left[\dfrac{d^2 f(t)}{dt^2}\right] = s^2 F(s) - s\ f(0\pm) - \dot{f}(0\pm)$
5	$L_{\pm}\left[\dfrac{d^n f(t)}{dt^n}\right] = s^n F(s) - \sum_{k=1}^{n} s^{n-k} \overset{(k-1)}{f}(0\pm)$ onde $\overset{(k-1)}{f}(t) = \dfrac{d^{k-1} f(t)}{dt^{k-1}}$
6	$L_{\pm}[\int f(t)dt] = \dfrac{F(s)}{s} + \dfrac{[\int f(t)dt]_{t=0\pm}}{s}$
7	$L_{\pm}[\iint f(t)\,dt\,dt] = \dfrac{F(s)}{s^2} + \dfrac{[\int f(t)\,dt]_{t=0\pm}}{s^2} + \dfrac{[\iint f(t)\,dt\,dt]_{t=0\pm}}{s}$
8	$L_{\pm}[\int \cdots \int f(t)(dt)^n\ ] = \dfrac{F(s)}{s^n} + \sum_{k=1}^{n} \dfrac{1}{s^{n-k+1}} [\int \cdots \int f(t)(dt)^k\ ]_{t=0\pm}$
9	$L[\int_0^t f(t)dt] = \dfrac{F(s)}{s}$
10	$\int_0^\infty f(t)dt = \lim_{s \to 0} F(s) \quad se \int_0^\infty f(t)dt\ \ existe$
11	$L[e^{-at}\ f(t)] = F(s+a)$
12	$L[\ f(t-\alpha)1(t-\alpha)\ ] = e^{-\alpha s}\ F(s) \quad \alpha \geq 0$
13	$L[t\ f(t)] = -\dfrac{dF(s)}{ds}$
14	$L[t^2\ f(t)] = \dfrac{d^2 F(s)}{ds^2}$
15	$L[t^n\ f(t)] = (-1)^n \dfrac{d^n F(s)}{ds^n} \quad n = 1,2,3,\cdots$
16	$L\left[\dfrac{1}{t}\ f(t)\right] = \int_s^\infty F(s)ds$
17	$L\left[f\left(\dfrac{t}{a}\right)\right] = aF(as)$

<u>Expansão em Frações Parciais</u>

<u>1º) Polos reais e distintos</u>

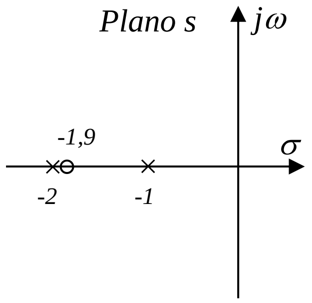
$$F(s)=\frac{B(s)}{A(s)}=\frac{K(s+z_1)(s+z_2)\dots(s+z_m)}{(s+p_1)(s+p_2)\dots(s+p_n)} \qquad (m \le n)$$

$$F(s)=\frac{a_1}{s+p_1}+\frac{a_2}{s+p_2}+\dots+\frac{a_n}{s+p_n}$$

$$a_k=\left[(s+p_k)\cdot\frac{B(s)}{A(s)}\right]\Bigg|_{s=-p_k} \quad \Rightarrow \quad \textit{Resíduo do polo } s=-p_k$$

Exemplo:

$$F(s)=\frac{s+1,9}{(s+2)(s+1)}=\frac{a_1}{s+2}+\frac{a_2}{s+1}$$



$$a_1=\frac{s+1,9}{s+1}\Bigg|_{s=-2}=\frac{-2+1,9}{-2+1}=0,1$$

$$a_2=\frac{s+1,9}{s+2}\Bigg|_{s=-1}=\frac{-1+1,9}{-1+2}=0,9$$

$$F(s)=\frac{0,1}{s+2}+\frac{0,9}{s+1} \quad \Rightarrow \quad \mathcal{L}^{-1}\left[F(s)\right]=f(t)=0,1\,e^{-2t}+0,9\,e^{-t}$$

<u>2º) Polos reais e múltiplos:</u>

$$F(s)=\frac{B(s)}{(s+p_k)^r}=\frac{a_1}{(s+p_k)^r}+\frac{a_2}{(s+p_k)^{r-1}}+\cdots+\frac{a_r}{s+p_k}$$

$$a_1=\left[(s+p_k)^r\;F(s)\right]\Big|_{s=-p_k}$$

$$a_2=\frac{d}{ds}\left[(s+p_k)^r\;F(s)\right]\Big|_{s=-p_k}$$

$$\vdots$$

$$a_r=\frac{1}{(r-1)!}\frac{d^{r-1}}{ds^{r-1}}\left[(s+p_k)^r\;F(s)\right]\Big|_{s=-p_k}$$

Exemplo: $F(s)=\dfrac{2s+3}{(s+1)^2}=\dfrac{a_1}{(s+1)^2}+\dfrac{a_2}{s+1}$

$$a_1=2s+3\big|_{s=-1}=-2+3=1$$

$$a_2=\frac{d}{ds}(2s+3)=2 \qquad \textit{ou} \qquad \begin{cases} 2s+3=a_1+a_2s+a_2 \\ a_2=2 \\ a_1+a_2=3 \Rightarrow a_1=1 \end{cases}$$

$$F(s)=\frac{1}{(s+1)^2}+\frac{2}{s+1} \quad \overset{\mathcal{L}^{-1}}{\Rightarrow} \quad f(t)=t\,e^{-t}+2\,e^{-t}=e^{-t}\,(2+t)$$

3º) Raízes Complexas

$$F(s)=\frac{B(s)}{(s+\sigma+j\omega)(s+\sigma-j\omega)}=\frac{a_1}{s+\sigma+j\omega}+\frac{a_2}{s+\sigma-j\omega}$$

Exemplo: $$F(s)=\frac{2}{s^2+2s+5}=\frac{a_1}{(s+1+j2)}+\frac{a_2}{(s+1-j2)}$$

$$a_1=\left.\frac{2}{(s+1-j2)}\right|_{s=-1-j2}=\frac{2}{-1-j2+1-j2}=-\frac{2}{j4}=j\,{}^1\!/_2$$

$$a_2=\left.\frac{2}{(s+1+j2)}\right|_{s=-1+j2}=\frac{2}{-1+j2+1+j2}=\frac{2}{j4}=-j\,{}^1\!/_2=a_1^* \quad \Rightarrow \text{ conjugado de } a_1$$

$$F(s)=\frac{j\,{}^1\!/_2}{s+1+j2}-\frac{j\,{}^1\!/_2}{s+1-j2} \quad \overset{\mathcal{L}^{-1}}{\Rightarrow}$$

$$f(t)=j\frac{1}{2}e^{(-1-j2)t}-j\frac{1}{2}e^{(-1+j2)t}$$

$$f(t)=j\frac{1}{2}e^{-t}\;e^{-j2t}-j\frac{1}{2}e^{-t}\;e^{j2t}=j\frac{1}{2}e^{-t}(e^{-j2t}-e^{j2t}) \qquad x\,\frac{j}{j}$$

$$f(t)=e^{-t}\left(\frac{e^{j2t}-e^{-j2t}}{2j}\right)=e^{-t}\;sen\,2t$$

Obs.: $s^2+2s+5=(s+1+j2)\,(s+1-j2)=(s+1)^2+2^2$

$$\cos\,\theta=\frac{e^{j\theta}+e^{-j\theta}}{2} \qquad\qquad sen\,\theta=\frac{e^{j\theta}-e^{-j\theta}}{2j}$$

Transformações de Diagramas de Blocos

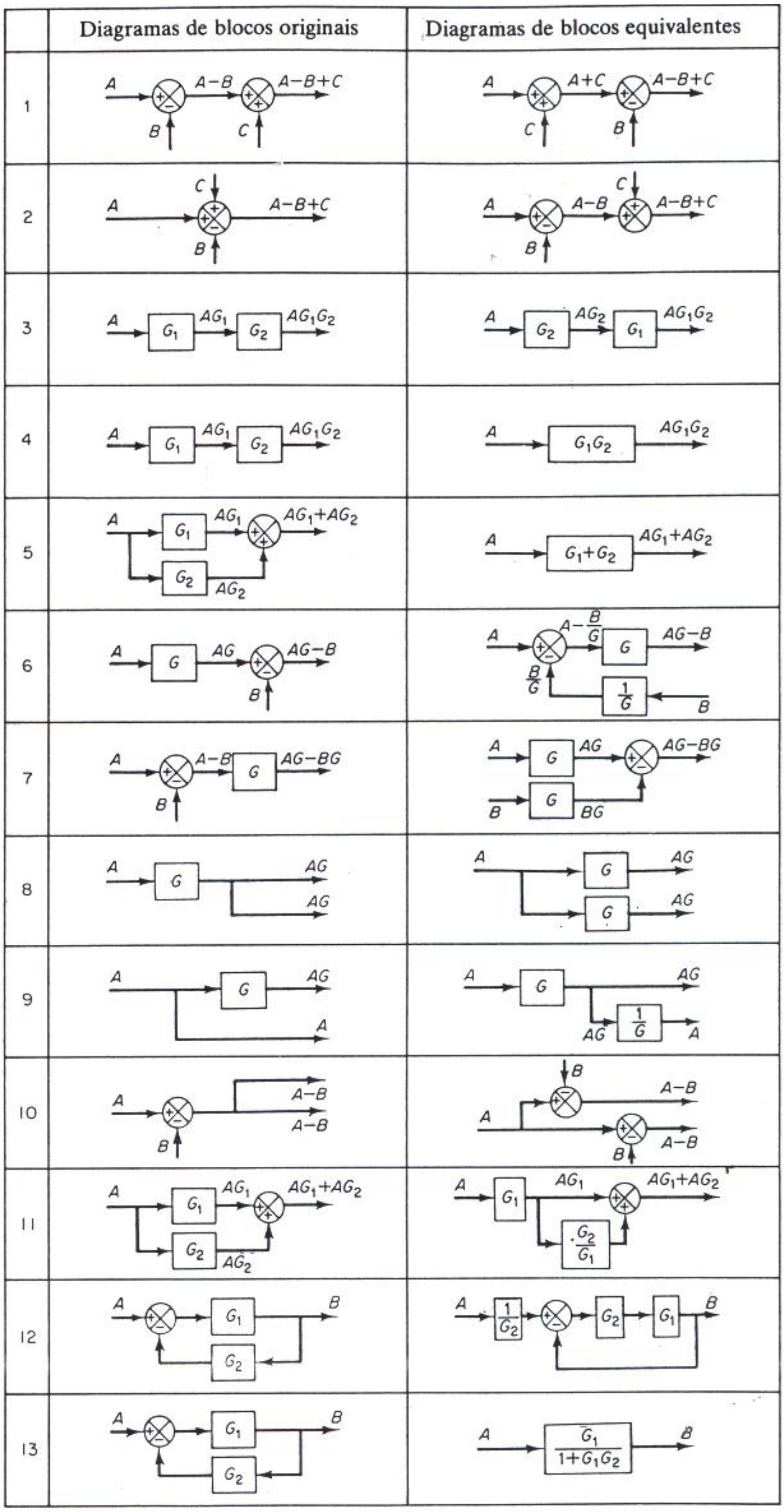


Tabela 5-1 Circuitos com amplificadores operacionais que podem ser usados como compensadores

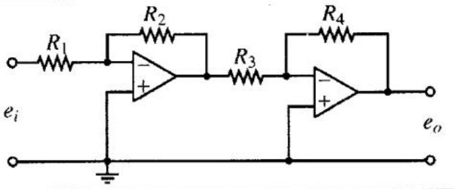
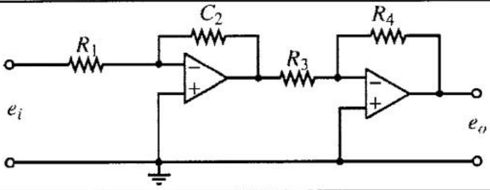
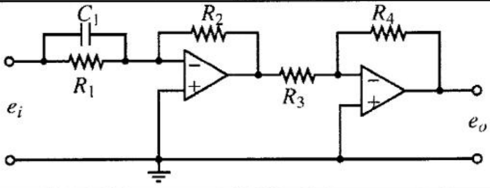
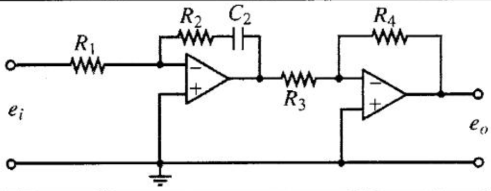
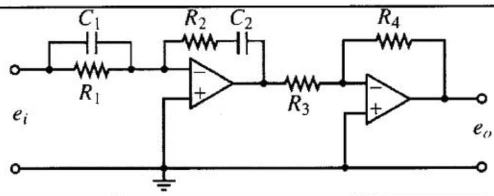
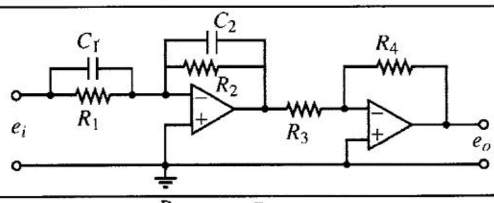
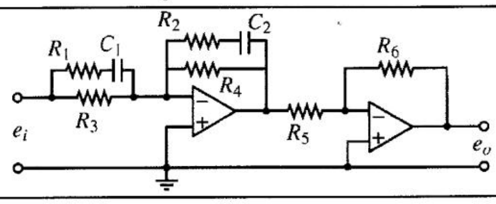
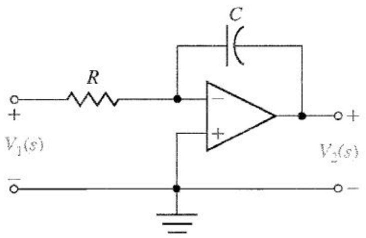
	Ação de controle	$G(s) = \dfrac{E_o(s)}{E_i(s)}$	Circuitos com amplificadores operacionais
1	P	$\dfrac{R_4}{R_3}\dfrac{R_2}{R_1}$	
2	I	$\dfrac{R_4}{R_3}\dfrac{1}{R_1C_2s}$	
3	PD	$\dfrac{R_4}{R_3}\dfrac{R_2}{R_1}(R_1C_1s+1)$	
4	PI	$\dfrac{R_4}{R_3}\dfrac{R_2}{R_1}\dfrac{R_2C_2s+1}{R_2C_2s}$	
5	PID	$\dfrac{R_4}{R_3}\dfrac{R_2}{R_1}\dfrac{(R_1C_1s+1)(R_2C_2s+1)}{R_2C_2s}$	
6	Avanço ou atraso de fase	$\dfrac{R_4}{R_3}\dfrac{R_2}{R_1}\dfrac{R_1C_1s+1}{R_2C_2s+1}$	
7	Avanço - Atraso de Fase	$\dfrac{R_6}{R_5}\dfrac{R_4}{R_3}\dfrac{[(R_1+R_3)C_1s+1](R_2C_2s+1)}{(R_1C_1s+1)[(R_2+R_4)C_2s+1]}$	

Tabela 2.5 Funções de Transferência de Circuitos e Elementos Dinâmicos

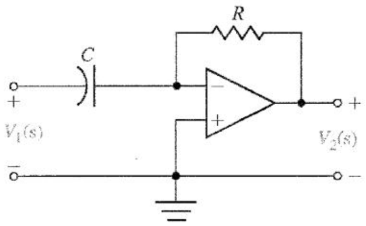
Elemento ou Sistema	G(s)

1. Circuito integrador, filtro



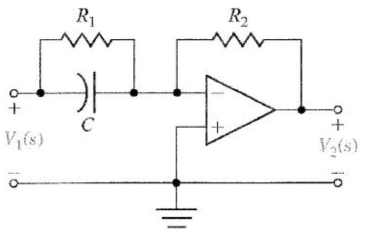
$$\frac{V_2(s)}{V_1(s)} = -\frac{1}{RCs}$$

2. Circuito diferenciador



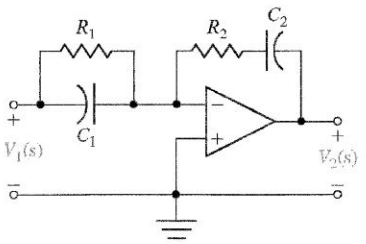
$$\frac{V_2(s)}{V_1(s)} = -RCs$$

3. Circuito diferenciador



$$\frac{V_2(s)}{V_1(s)} = -\frac{R_2(R_1Cs + 1)}{R_1}$$

4. Filtro integrador



$$\frac{V_2(s)}{V_1(s)} = -\frac{(R_1C_1s + 1)(R_2C_2s + 1)}{R_1C_2s}$$

(continua)

TABELA 2.7 Funções de Transferência de Elementos Dinâmicos e de Circuitos

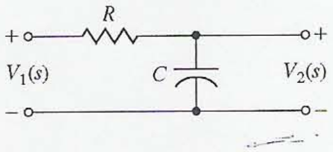
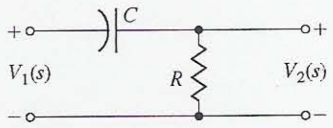
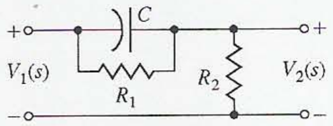
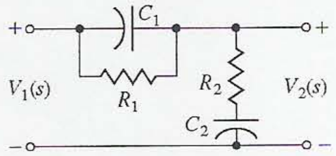
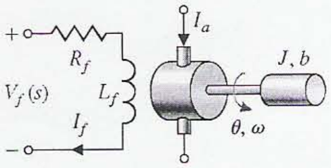
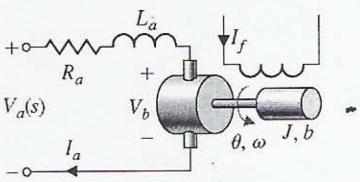
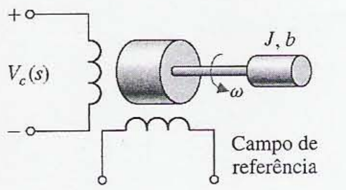
Elemento ou Sistema	G(s)
1. Circuito integrador, filtro 	$$\frac{V_2(s)}{V_1(s)} = \frac{1}{RCs + 1}$$
2. Circuito diferenciador 	$$\frac{V_2(s)}{V_1(s)} = \frac{RCs}{RCs + 1}$$
3. Circuito diferenciador 	$$\frac{V_2(s)}{V_1(s)} = \frac{s + 1/R_1 C}{s + (R_1 + R_2)/R_1 R_2 C}$$
4. Circuito de filtro de avanço-atraso de fase  $\tau_a = R_1 C_1$ $\tau_b = R_2 C_2$ $\tau_{ab} = R_1 C_2$ $\tau_1 \tau_2 = \tau_a \tau_b$ $\tau_1 + \tau_2 = \tau_a + \tau_b + \tau_{ab}$	$$\frac{V_2(s)}{V_1(s)} = \frac{(1 + s\tau_a)(1 + s\tau_b)}{\tau_a \tau_b s^2 + (\tau_a + \tau_b + \tau_{ab})s + 1} = \frac{(1 + s\tau_a)(1 + s\tau_b)}{(1 + s\tau_1)(1 + s\tau_2)}$$
5. Motor CC, controlado pelo campo, atuador rotativo 	$$\frac{\theta(s)}{V_f(s)} = \frac{K_m}{s(Js + b)(L_f s + R_f)}$$
6. Motor CC, controlado pela armadura, atuador rotativo 	$$\frac{\theta(s)}{V_a(s)} = \frac{K_m}{s[(R_a + L_a s)(Js + b) + K_b K_m]}$$

TABELA 2.7 *Continuação*

Elemento ou Sistema	***G*(*s*)**

7. Motor CA, bifásico com enrolamento de controle, atuador rotativo

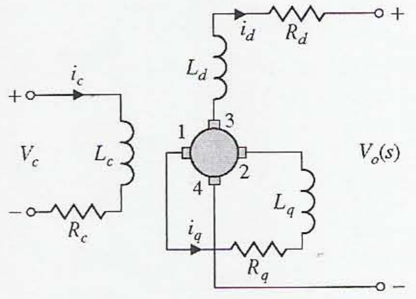


$$\frac{\theta(s)}{V_c(s)} = \frac{K_m}{s(\tau s + 1)}$$

$$\tau = J/(b - m)$$

m = inclinação (normalmente negativa) da curva de torque-velocidade linearizada

8. Amplidina, amplificador de tensão e de potência



$$\frac{V_o(s)}{V_c(s)} = \frac{(K/R_c R_q)}{(s\tau_c + 1)(s\tau_q + 1)}$$

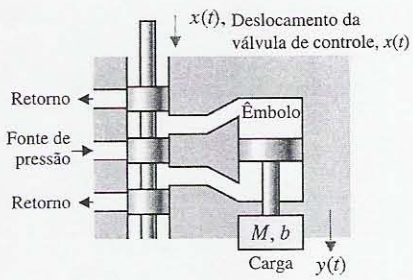
$$\tau_c = L_c/R_c, \quad \tau_q = L_q/R_q$$

Para o caso de operação em vazio (sem carga) $i_d \simeq 0$, $\tau_c \simeq \tau_q$,

$0{,}05\text{ s} < \tau_c < 0{,}5\text{ s}$

$V_{12} = V_q$, $V_{34} = V_d$

9. Atuador hidráulico



$$\frac{Y(s)}{X(s)} = \frac{K}{s(Ms + B)}$$

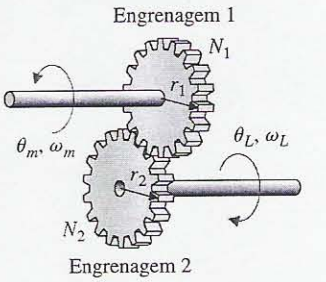
$$K = \frac{Ak_x}{k_p}, \quad B = \left(b + \frac{A^2}{k_p}\right)$$

$$k_x = \left.\frac{\partial g}{\partial x}\right|_{x_0}, \quad k_p = \left.\frac{\partial g}{\partial P}\right|_{P_0},$$

$g = g(x, P)$ = vazão

A = área do êmbolo

10. Trem de engrenagens, transformador de rotação



Relação de engrenagens $= n = \dfrac{N_1}{N_2}$

$$N_2\theta_L = N_1\theta_m, \quad \theta_L = n\theta_m$$

$$\omega_L = n\omega_m$$

TABELA 2.7 *Continuação*

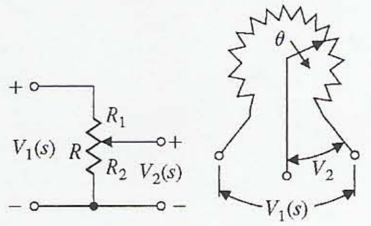
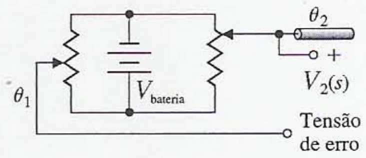
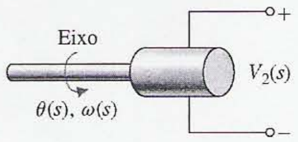
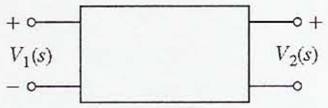
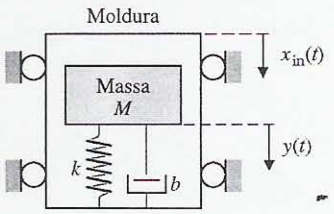
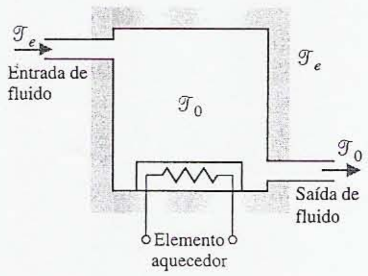
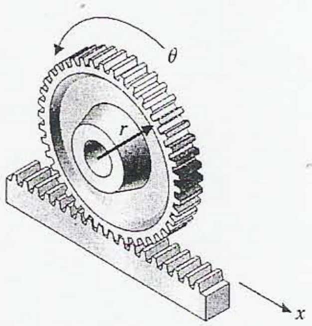
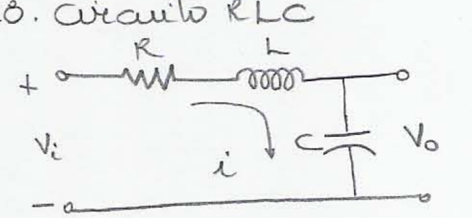
Elemento ou Sistema	$G(s)$
11. Potenciômetro, controle de tensão  	$\dfrac{V_2(s)}{V_1(s)} = \dfrac{R_2}{R} = \dfrac{R_2}{R_1 + R_2}$ $\dfrac{R_2}{R} = \dfrac{\theta}{\theta_{máx}}$
12. Ponte potenciométrica para detecção de erro  	$V_2(s) = k_s(\theta_1(s) - \theta_2(s))$ $V_2(s) = k_s\theta_{erro}(s)$ $k_s = \dfrac{V_{bateria}}{\theta_{máx}}$
13. Tacômetro, sensor de velocidade  	$V_2(s) = K_t\omega(s) = K_t s\theta(s)$; K_t = constante
14. Amplificador CC  	$\dfrac{V_2(s)}{V_1(s)} = \dfrac{k_a}{s\tau + 1}$ R_o = resistência de saída C_o = capacitância de saída $\tau = R_o C_o$, $\tau \ll 1$ e é freqüentemente desprezada nos amplificadores para servomecanismos
15. Acelerômetro, sensor de aceleração  	$x_o(t) = y(t) - x_{in}(t)$, $\dfrac{X_o(s)}{X_{in}(s)} = \dfrac{-s^2}{s^2 + (b/M)s + k/M}$. Para oscilações de freqüência baixa, onde $\omega < \omega_n$, $\dfrac{X_o(j\omega)}{X_{in}(j\omega)} \simeq \dfrac{\omega^2}{k/M}$

TABELA 2.7 ***Continuação***

Elemento ou Sistema	G(s)
16. Sistema térmico de aquecimento 	$\frac{\mathcal{T}(s)}{q(s)} = \frac{1}{C_t s + (QS + 1/R)}$, onde $\mathcal{T} = \mathcal{T}_o - \mathcal{T}_e$ = diferença de temperatura devida ao processo térmico C_t = capacitância térmica Q = vazão de fluido = constante S = calor específico da água R_t = resistência térmica do isolamento $q(s)$ = vazão do fluxo térmico do elemento aquecedor
17. Cremalheira e pinhão 	$x = r\theta$ converte movimento angular em movimento linear

Sistemas de 2ª ordem

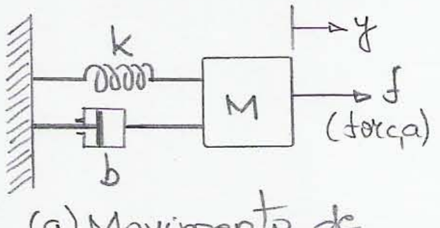
18. Circuito RLC



$$v_i(t) = R\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(t)\,dt$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1/LC}{s^2 + R/L\, s + 1/LC}$$

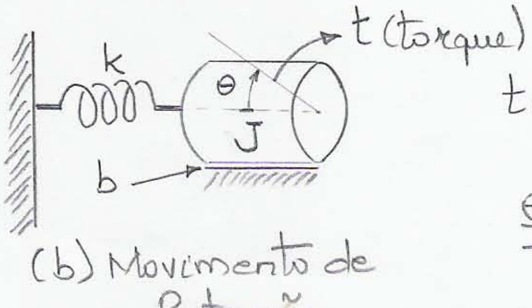
19. Sistema Massa-Mola-Amortecedor



(a) Movimento de Translação

$$f(t) = M\frac{d^2y(t)}{dt} + b\frac{dy(t)}{dt} + k\,y(t)$$

$$\frac{Y(s)}{F(s)} = \frac{1/M}{s^2 + b/M\, s + k/M}$$



(b) Movimento de Rotação

$$t(t) = J\frac{d^2\theta(t)}{dt} + b\frac{d\theta(t)}{dt} + k\,\theta(t)$$

$$\frac{\Theta(s)}{T(s)} = \frac{1/J}{s^2 + b/J\, s + k/J}$$