Solos e Rochas - Revista Brasileira de Geotecnia, Janeiro-Abril, v. 27, n. 1, p. 57-68, 2004.

Evaluation of Soil-Water Characteristic Curves for Problems of Unsaturated flow in Soils from Brazil

Avaliação de Curvas Características com Aplicações a Problemas de Fluxo em Solos Não Saturados no Brasil

Previsión de las Curvas Características Aplicadas a Problemas del Flujo en Suelos no Saturados en Brasil

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Abstract. The relationship between the soil-water content and suction is commonly referred to as the soil-water characteristic curve (SWCC). The SWCC is a useful tool for predicting the engineering behavior of unsaturated soils. Several equations have been proposed in the literature for mathematically reproducing the experimental behavior of the SWCC. All equations require the definition of some curve fitting parameters, besides the bubbling pressure and the saturated and residual volumetric water contents. Previous studies by the authors have indicated that the equations proposed by Gardner, van Genuchten and Fredlund & Xing provide a good estimate of the SWCC for soils from Brazil. A computer program based on Genetic Algorithm was coupled to an Excel spreadsheet, to optimize the evaluation of curve fitting parameters. This methodology, described in the paper, revealed to be an easy, quick and effective tool for predicting SWCC. The influence of an accurate prediction of the SWCC, as well as hydraulic conductivities, on the flow processes through unsaturated soil profiles was also evaluated. Experimental data at three different depths, from a same site in Brazil, were used in this study. Changes of both parameters with depth produced significant differences in flow simulations.

Resumo: A relação entre teor de umidade e sucção, usualmente denominada de curva característica (SWCC), é um parâmetro muito utilizado na previsão do comportamento de solos não saturados. Várias equações foram propostas na literatura na tentativa de reproduzir matematicamente esta relação. Todas equações requerem a definição de alguns parâmetros de ajuste, além da sucção de entrada de ar e dos teores de umidade residual e saturado. Estudos anteriores, publicados pelos autores, mostraram que as equações propostas Gardner, van Genuchten e Fredlund & Xing forneciam bom ajustes das curvas características de solos brasileiros. Este trabalho apresenta uma metodologia de otimização do processo de determinação dos parâmetros de ajuste. Esta metodologia envolve o acoplamento de um programa, baseado em Algoritmos Genéticos, a uma planilha Excel, especialmente desenvolvida para este fim. Os resultados mostraram que este acoplamento é simples e eficaz A influência de uma previsão adequada não só da curva característica, mas também da condutividade hidráulica no desenvolvimento de regimes de fluxos através de solos inicialmente não saturados também foi avaliada. Foram utilizados resultados experimentais correspondentes a três diferentes profundidades de um campo experimental, no Brasil. A simulação do regime de fluxo mostrou que ambos parâmetros influenciam significativamente na previsão da distribuição das cargas hidráulicas.

Resumen: La relación entre la humedad del suelo y la succión, denominada de curva característica (SWCC), es un parámetro muy utilizado en la previsión de la conducta de suelos no saturados. Han sido propuestas varias ecuaciones, en la literatura, en la tentativa de reproducir matemáticamente su conducta experimental. Todas las ecuaciones requieren la definición de algunos parámetros de ajuste, además de la succión de entrada de aire e de las humedades residual y saturada. Estudios anteriores, publicados por los autores, han indicado que las ecuaciones propuestas por Gardner, van Genuchten y Fredlund & Xing ofrecen una buena estimación de las curvas características de suelos brasileños. Un programa basado en Algoritmos Genéticos ha sido acoplado a hoja de cálculo Excel, con el objetivo de optimizar la determinación de los parámetros de ajuste. Esta metodología, descrita en este trabajo, ha resultado en una reducción de los errores entre las curvas prevista y determinada experimentalmente. La influencia de una previsión adecuada de la SWCC e de la permeabilidad en el desarrollo del régimen de flujo a través del suelo no saturado también ha sido avaluada. Han sido utilizados resultados experimentales correspondientes a tres distintas profundidades de un campo experimental, en Brasil. La simulación del régimen de flujo se ha mostrado muy influenciada por cambios en el dios parámetros.

1. Introduction

Several numerical models for simulating flow behavior through unsaturated porous media have been proposed in the last decades (Bear & Verruijt 1987, Huyakorn & Pinder, 1983, Pinder & Gray, 1977). In these models it is a common practice to use as hydraulic parameters, the soil-water characteristic curve (SWCC), defined by the relationship between soil suction and volumetric water content, and the relative hydraulic conductivity, which is the ratio between the unsaturated and saturated hydraulic conductivities. The direct measurement of these parameters is time consuming and expensive. Therefore, different equations were proposed in the last decades to mathematically represent both experimental behavior of the SWCC and relative hydraulic conductivity.

All SWCC equations require definition of at least 2 independent parameters, besides some curve shape parameters: bubbling pressure (ψ_b) and saturated (θ_s) and residual (θ_r) volumetric water contents

Previous studies have evaluated the suitability of different SWCC equations for fitting experimental data of soils from Brazil (Gerscovich, 2001; Gerscovich & Sayao, 2002). In these studies, a manual search procedure was used for computing the independent equation parameters. The results have indicated that the equations proposed by Gardner, Van Genuchten and Fredlund & Xing provided a good estimate of the SWCC for soils from Brazil. Moreover, the experimental data revealed the variability of the SWCC within a soil profile, and, consequently, the difficulty in defining a single curve for describing an entire soil layer.

This paper describes a methodology to assess fitting parameters of SWCC equations, by coupling an optimization process, based on Genetic Algorithm, with an Excel spreadsheet. The variability SWCC is also discussed by comparing 1D flow simulation within a multiple layer and a single layer soil profile.

2. Genetic Algorithm

Genetic algorithms (GAs) are considered an artificial intelligence technology, inspired by Darwin's evolution theory.

GAs are primarily a search mechanism, in a predefined space, which gives a minimum or maximum solution of an equation. This technique is very useful in complex optimization problems, when several parameters or characteristics need to be combined to achieve the best solution.

There are some essential differences between GAs and other forms of optimization. Among others, genetic algorithms use a set of points to conduct a search, not only a single point on the problem space. This facility allows performing search in noisy spaces with local optimum points. Instead of relying on a single point to search through the space, the GAs look at many different areas of the problem space at once, and uses all this information to guide it. For a more complete discussion, see Goldberg, 1989.

The algorithm is based on an iterative process, which must be started with a set of solutions (represented by “chromosomes”), called “population”. This initial guess must in some way contain information about solution. This information may be a numerical value within boundary limits. As example, in binary encoding, a set of solution composed by 2 “chromosomes” could look like the representation shown in Figure 1.

Chromosome 1	1101100100110110
Chromosome 2	1101111000011110

Figure 1. Set of solution (“population”)

Solutions from one “population” are then taken and used to form a new one. When a new solution is to be created, two “parents” are chosen from the current population. The elected “parents” are selected according to their fitness; i.e., the more suitable they are the more chances they have to reproduce.

The new set of solution (new “population”) is initially generated by crossover operation, followed by mutation process.

The crossover algorithm choose some crossover point, and everything before this point is a copy from a first “parent” while everything after the crossover point is a copy from the second one. The resulting set of solution is called offspring.

The crossover rate can be set between 0.01 and 1.0, and reflects the mix of information from the previous “generation” that will be enclosed in the future set of solutions. A rate of 0.9, for example, means that roughly 90% of an offspring organism's values will come from the first parent and 10% will come from the second parent. A crossover rate of 1 means that no crossover will occur, so only clones of the parents will be evaluated. Figure 2 shows an example of a crossover operation with a rate of 0.3.

Chromosome 1	**11001 | 00100110110**
Chromosome 2	11011 | 11000011110
Offspring 1	**11001** | 11000011110
Offspring 2	11011 | **00100110110**

Figure 2. Crossover operation

After crossover process, mutation takes place. This step is performed to prevent that all solutions fall into a minimum (or maximum) local point, instead of achieving an optimum global solution. Mutation changes “offsprings” by randomly switching bits from 1 to 0 or from 0 to 1, in a rate that may be set to between 0.0 and 1.0. A higher mutation rate means that more bits are changed. When this rate is set equal to 1, the whole code is mutated, removing any crossover effect. Figure 3 shows an example, in binary encoding, with a mutation rate equal to 0.3.

Original offspring 1	110**0**111000011110
Original offspring 2	110110**0**1001101**1**0
Mutated offspring 1	110**1**111000011110
Mutated offspring 2	110110**1**1001101**1**0

Figure 3. Offspring mutation

This whole process is repeated until some condition (minimum or maximum) is satisfied. The stopping conditions may be associated to an error criterion, to a maximum number of trials or to a computer processing time.

Further details on GA’s search process may be found in Goldberg (1989).

2.1 Evolver program

Evolver is a computer program, conceived as a Microsoft Excel macro capability that uses Genetic Algorithms (GAs) to perform optimization processes.

The program employs both crossover and mutation operators, with a steady-state approach. This means that only one organism is replaced at time, rather than substituting an entire set of solution. This steady state technique has shown to be more effective or even better than general replacement method.

The Excel spreadsheet must be prepared with the mathematical model. Initial guesses of model variables are inputted in the so-called adjustable cells and represent the first “population”.

Three types of variable constraints may be specified. Range constraint defines minimum and maximum possible values. Hard constraint establishes a specific value and soft constraint set up some conditions to be met as much as possible.

The model output is given in the target cell. This cell contains a formula, which depends directly or through a series of calculations on the adjustable cells.

There are three different stopping conditions during the optimization process. Trial option stops the program when a given number of trials is carried out. Minute option is related to a preset time interval. Change in last option stops the process when the improvement in the target cell is less than a specified amount (change criterion).

Further details in Evolver User’s Guide are presented by Palisade Corporation (2002).

3. Soil-Water Characteristic Curve Equations

The soil-water characteristic curve (SWCC) is defined as the relationship between soil suction and volume of water in the soil pores. The water content can be established in volumetric (θ) or gravimetric (ω) terms or, alternatively expressed by the degree of saturation (S). The volumetric water content is usually adopted and the SWCC is referred to as the soil-water retention curve. The volumetric water content (θ) is defined by the ratio between pore water volume and total volume, and is equivalent to porosity (n) at full saturation.

Soil matric suction (ψ) is defined as the difference between pore-air pressure (u_a) and pore-water pressure (u_w). Total suction (ψ_t) is equal to the sum of osmotic suction and matric suction. For practical engineering applications it can be assumed that the total suction is equal to osmotic suction for high values of soil suction (above 1500kPa).

The usual S-shape of the SWCC may be defined by four parameters: saturated volumetric water content (θ_s); residual volumetric water content (θ_r); air-entry value or bubbling pressure (ψ_b) and volumetric water retention capacity ($\Delta\psi/\Delta\theta$), shown in Figure 4.

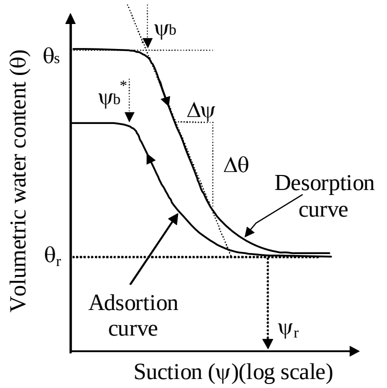


Figure 4. SWCC shape parameters

In a gradually wetting process of an initially dry soil (adsorption curve), the water contents are lower than the values corresponding to the drying curve (desorption curve), at any suction value. As a result, full saturation condition is rarely achieved in wetting processes. This hysteretic pattern is mainly attributed to geometric non-uniformities of the interconnected pores and/or to entrapped air. Soil structure changes due to swelling or shrinking phenomena are also partially responsible for this response (Hillel 1971). It has been experimentally observed that the difference in θ-values increases with the percentage of coarse particles (Smith & Browning 1942; Wilson et al. 1981).

The bubbling pressure (ψ_b) defines the soil suction at which water in the largest pores starts to drain. ψ_b value is relatively small and depends on the pore-size of the soil. It is expected a range of ψ_b = 0.2 to 7.5kPa for coarse to fine sands, ψ_b = 7 to 25kPa for silty soils, and a $\psi_b > 25$kPa for clays (Aubertin et al. 1998). The bubbling pressure can be graphically estimated, as shown in Figure 4. Aubertin et al. (1998) proposed that ψ_b may be considered as the matric suction corresponding to $\theta = 0.9\ \theta_s$. At this volumetric water content, the authors suggest that continuous channels are created within the soil. This approach results in ψ_b values 25% higher than those obtained by the graphical method.

The residual volumetric water content (θ_r) is a lower limit; beyond this limit an increase in matric suction does not reduce significantly water content.

A number of equations are available in the literature to mathematically represent the SWCC data. Most equations (Table 1) are based on the assumption that the shape of this curve is dependent upon pore size distribution (Gardner 1958; Brooks & Corey 1964; Farrel & Larson; 1972; van Genuchten 1980; William et al.1983; Saxton et al. 1986; Haverkamp & Parlange 1986; McKee & Bumb 1987; Fredlund & Xing 1994, Aubertin et al, 1998). This assumption implicitly considers a spherical shape for the water-air meniscus in the pores. It is therefore assumed a cylinder shape for the interconnected channels within the soil. The parameters for these equations are calibrated by linear regression of experimental data.

Other equations assume that the SWCC can be directly estimated from the grain size distribution and physical properties of soils (Ghosh 1980; Rawls & Brakensiek 1989), as indicated in Table 2. These simple propositions are convenient in engineering practice, because grain size distributions can be determined in all conventional soil laboratories. However, these procedures disregard stress state, soil structure, compaction water content, compaction energy and mineralogy that play a major role in defining the shape of the SWCC and influence the flow behavior in unsaturated soils.

Previous studies have examined the suitability of 14 different SWCC equations for fitting experimental data of soils from Brazil (Gerscovich, 2001; Gerscovich and Sayao, 2002). An Excel spreadsheet for curve fitting was specially developed, and 11 different soils were analyzed. In these studies, a manual search process was used to estimate equation's fitting parameters. Each equation parameter was varied independently. Theses studies have revealed that the equations proposed by Gardner (1958), van Genuchten (1980), and Fredlund & Xing (1994) provided best fits of Brazilian soils experimental data. Gardner's equation requires the smallest number of unknown parameters (α, η). The proposition by van Genuchten (1980) is comparable to the previous one, but requires an additional exponent parameter. Due to this similarity, this proposition was disregarded in this paper. Fredlund & Xing's equation involves 3 unknown parameters (*a*, *m* and *n*).

4. Prediction of SWCC Equation Parameters with Genetic Algorithm

Evolver computer program was used to estimate SWCC equation parameters for fitting experimental data from 3 different depths of a same site, in the city of São Carlos, Brazil.

4.1 Experimental Data

Machado & Vilar (1998) carried out water-retention laboratory tests to determine the SWCC at three different depths at São Carlos site, São Paulo. At this site, the soil profile is composed by a 6.5m thick sedimentary soil overlying a 13.5m thick residual soil.

The undisturbed samples were extracted from 3m, 5m and 8m deep. The laboratory tests were performed with tension plate device, for low soil suction range (ψ < 13kPa), and pressure cell, for higher values (up to 350kPa). Sixteen experimental data were obtained for each depth.

The SWCC shape parameters were evaluated from the experimental data and are presented in Table 3, with soil characterization.

The saturated volumetric water content (θ_s) was assumed as being equal to 95% of porosity (n). At full saturation condition, the ratio θ_s /n should be 1.0. However, during wetting processes complete saturation is rarely achieved, as a result of geometric non-uniformities of the interconnected pores and/or to entrapped air. It is likely that all samples undergo both wetting and drying paths, since in situ soil suction ranged from about 8 to 30kPa.

In all tests, the residual soil suction (ψ_r) and the air-entry values (ψ_b) were assumed to be equal to 10^4kPa and 1kPa, respectively These assumptions were considered reasonable, despite different percentages of granular and fine materials of soil samples (Gerscovich, 2001).

4.2 Results

An Excel spreadsheet for curve fitting was specially developed to assess Gardner's and Fredlund & Xing's equation parameters. The quality of curve fitting was measured by an error criterion (ε) defined by the following equation.

$$\varepsilon = \frac{\sqrt{\sum(\theta_i - \hat{\theta}_i)^2}}{\sqrt{\sum(\hat{\theta}_i)^2}} \qquad (1)$$

where θ_i and $\hat{\theta}_i$ are, respectively the predicted and measured volumetric water contents.

Figure 5 shows Excel spreadsheet developed for Gardner's equation. The four initial columns are related to predicted values. The experimental data are listed in the 2 last columns. The target cell corresponds to the computed error and the parameters α and η are set as adjustable cells .

The residual volumetric water content (θ_r) was also assumed as an adjustable parameter. This approach was used in Gardner's simulations, in an attempt to reduce uncertainties associated to its experimental evaluation. The saturated volumetric water content θ_s is usually determined experimentally, whereas θ_r is not easily quantified. In the present paper, residual volumetric water content (θ_r^*) was allowed to vary between zero and saturated moisture condition (θ_s).

Figure 5 also shows Evolver settings window, where GAs required parameters are inputted (target and adjustable cells, constraints, GA operators, and stopping conditions). As shown in Table 4, the variable constraints were allowed to vary within a wide search space. Crossover and mutation rates were set equal to 0.5 and 0.06, respectively. The stopping condition was defined by a preset time equal to 10 min.

At the end of the optimization process, the best solutions were automatically placed in the adjustable and target cells. More detailed information about the whole process could also be obtained by accessing output windows. Figure 6 shows an output window example, that summarizes total number of trials, original; and best values computed for all adjustable and target cells, etc.

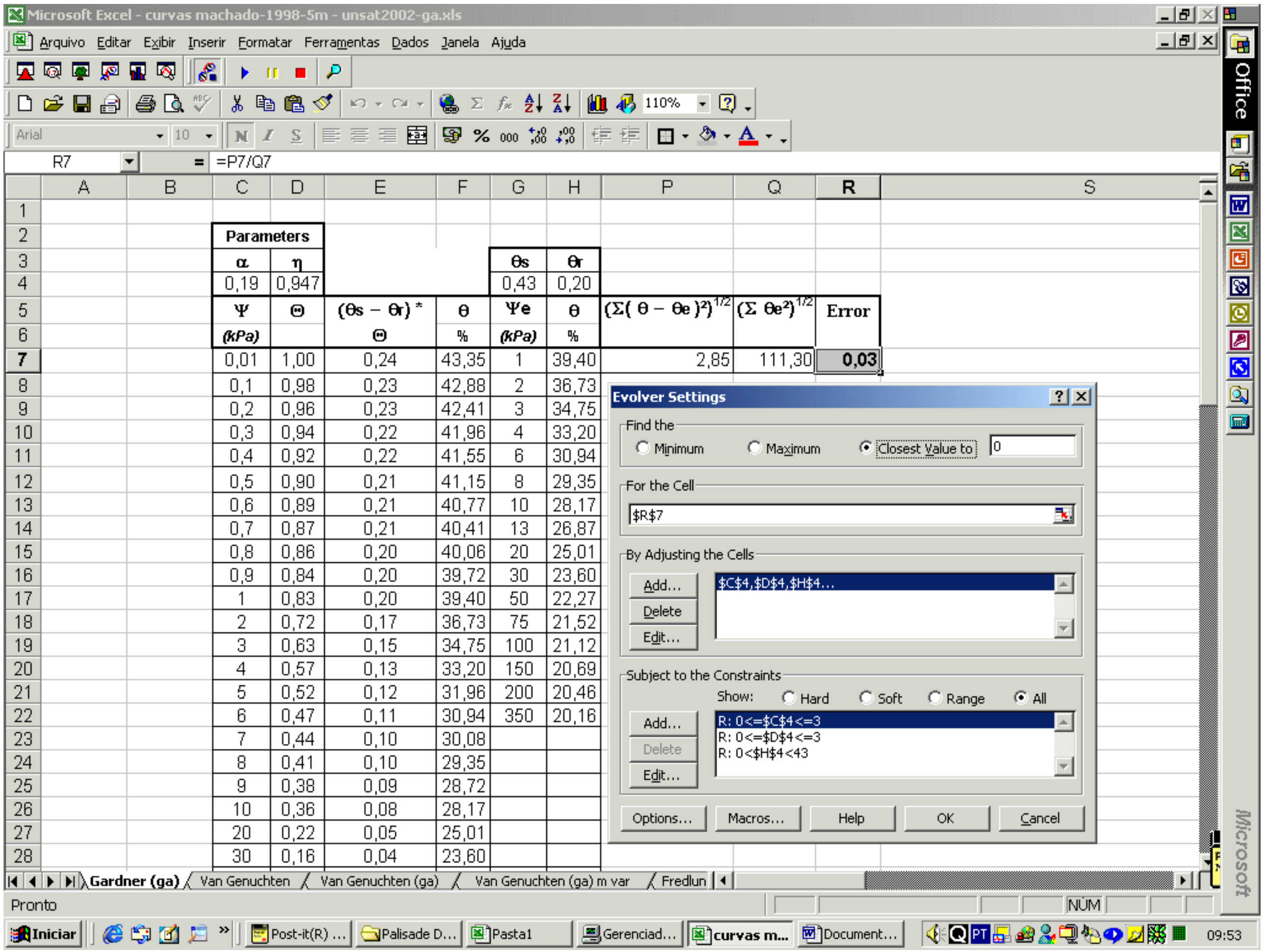
The resulting SWCC parameters and computed errors are listed in Table 5, with the corresponding values obtained from a manual search procedure (Gerscovich & Sayão, 2002). It is also presented the elapsed time for convergence.

In the manual search procedure each equation parameter was independently varied. This approach was time consuming since there were no physical meanings for most of the equation parameters.

Numerical search process has revealed to be more effective with Gardner's equation. An average 50% error reduction is observed in 3 m deep soil sample. This improvement resulted in a better curve fitting at low suction range, as shown in Figure 7.

All numerical analyses were carried out at least for 10min, to observe convergence progression. During this period almost 3.5x10^{6} trials were processed. However, as shown Figure 8, a sharp error reduction occurs within the first initial minute. After this period no further improvement is observed.

The residual volumetric water content (θ_r^*) computed by Evolver resulted in θ_r^*-values slightly higher than the ones based on the experimental data (Table 3). θ_r free search has shown to be a convenient approach, since it reduces the number of required parameters to be evaluated from experimental data and provide a best curve fitting.



R7 = =P7/Q7

	C	D	E	F	G	H	P	Q	R
1									
2	Parameters								
3	α	η			θs	θr			
4	0,19	0,947			0,43	0,20			
5	Ψ	Θ	(θs − θr) *	θ	Ψe	θ	$(\Sigma(\theta - \theta e)^2)^{1/2}$	$(\Sigma\ \theta e^2)^{1/2}$	Error
6	(kPa)		Θ	%	(kPa)	%			
7	0,01	1,00	0,24	43,35	1	39,40	2,85	111,30	0,03
8	0,1	0,98	0,23	42,88	2	36,73			
9	0,2	0,96	0,23	42,41	3	34,75			
10	0,3	0,94	0,22	41,96	4	33,20			
11	0,4	0,92	0,22	41,55	6	30,94			
12	0,5	0,90	0,21	41,15	8	29,35			
13	0,6	0,89	0,21	40,77	10	28,17			
14	0,7	0,87	0,21	40,41	13	26,87			
15	0,8	0,86	0,20	40,06	20	25,01			
16	0,9	0,84	0,20	39,72	30	23,60			
17	1	0,83	0,20	39,40	50	22,27			
18	2	0,72	0,17	36,73	75	21,52			
19	3	0,63	0,15	34,75	100	21,12			
20	4	0,57	0,13	33,20	150	20,69			
21	5	0,52	0,12	31,96	200	20,46			
22	6	0,47	0,11	30,94	350	20,16			
23	7	0,44	0,10	30,08					
24	8	0,41	0,10	29,35					
25	9	0,38	0,09	28,72					
26	10	0,36	0,08	28,17					
27	20	0,22	0,05	25,01					
28	30	0,16	0,04	23,60					

Evolver Settings

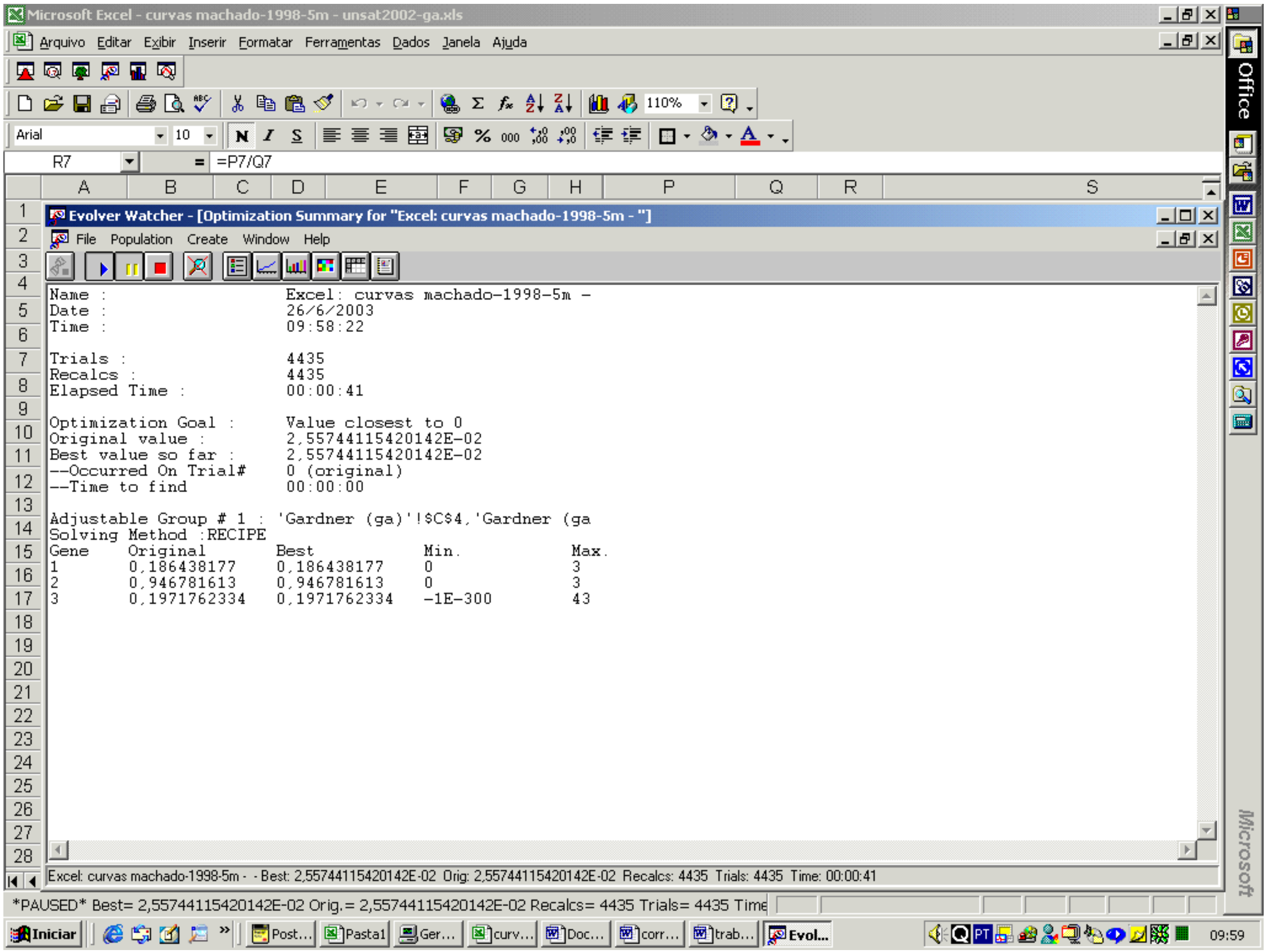
Find the
○ Minimum ○ Maximum ● Closest Value to 0

For the Cell
$R$7

By Adjusting the Cells
$C$4,$D$4,$H$4...

Subject to the Constraints
Show: ○ Hard ○ Soft ○ Range ● All
R: 0<=$C$4<=3
R: 0<=$D$4<=3
R: 0<$H$4<43

Figure 5. Excel spreadsheet (Gardner's equation)



```
Name :                        Excel: curvas machado-1998-5m -
Date :                        26/6/2003
Time :                        09:58:22

Trials :                      4435
Recalcs :                     4435
Elapsed Time :                00:00:41

Optimization Goal :           Value closest to 0
Original value :              2,55744115420142E-02
Best value so far :           2,55744115420142E-02
--Occurred On Trial#          0 (original)
--Time to find                00:00:00

Adjustable Group # 1 :  'Gardner (ga)'!$C$4,'Gardner (ga
Solving Method :RECIPE
Gene     Original       Best          Min.          Max.
1        0,186438177    0,186438177   0             3
2        0,946781613    0,946781613   0             3
3        0,1971762334   0,1971762334  -1E-300       43
```

```
Excel: curvas machado-1998-5m - - Best: 2,55744115420142E-02 Orig: 2,55744115420142E-02 Recalcs: 4435 Trials: 4435 Time: 00:00:41
```

```
*PAUSED* Best= 2,55744115420142E-02 Orig.= 2,55744115420142E-02 Recalcs= 4435 Trials= 4435 Time
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Figure 6. Evolver detailed output

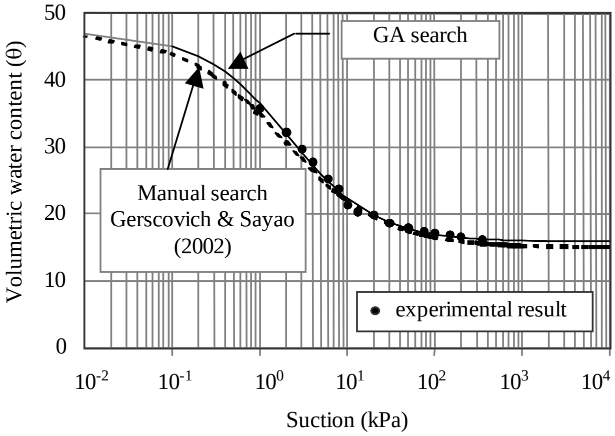


Figure 7. Manual process *vs* GA process – Gardner's Equation (z=3m)

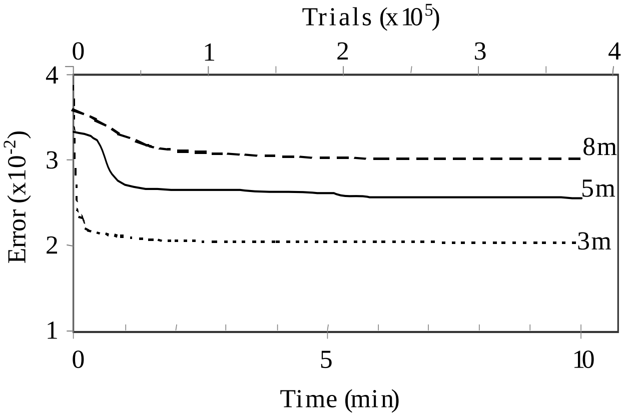


Figure 8. Computed error *vs* time - Gardner's equation

5. Influence of SWCC Variability on Unsaturated Flow Prediction

Figure 9 shows Fredlund and Xing's equation curve predictions, using GA, for three different depths at São Carlos site. The curves are distinct, in spite the two superficial samples refer to the same material. This variability of the SWCC may suggest that any unsaturated flow simulation, that uses water retention capacity ($\Delta\psi/\Delta\theta$), defined by the tangent of the SWCC, should consider specific curves for the different soil layers.

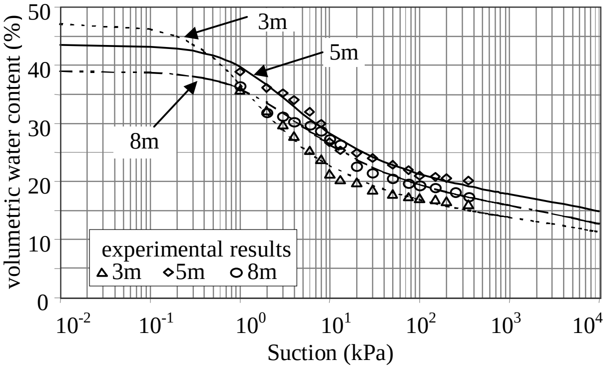


Figure 9. Fredlund & Xing's prediction using GA

The influence of the variability of the SWCC within depth on flow modeling was then tested in a 1D transient flow simulation within São Carlos site profile.

At this site, the soil profile is composed by a 6.5m thick sedimentary soil overlying a 13.5m thick residual soil, as shown in Figure 10. The sedimentary soil was divided in two regions to account for the differences of the SWCC at 3m and 5m deep.

The hydraulic conductivity and its variation with soil suction were computed in accordance to Gardner's proposition:

$$k = k_{sat} e^{\alpha_1 \psi} \qquad (2)$$

where, k_{sat} = saturated hydraulic conductivity; ψ = matric suction and α_1 = equation parameter.

Hydraulic conductivity field tests, carried out in a Brazilian residual soil, provided an average α_1 value equal to $1m^{-1}$ (Gerscovich, 1994). This value was used for both soils, and resulted in a relationship between relative hydraulic conductivity (k/k_{sat}) and soil suction, which is presented in Figure 11. The saturated hydraulic conductivity of the residual soil was assumed to be equal to $1x10^{-3}$ cm/s. In the sedimentary soil, k_{sat} was considered two times greater, due to the greater void ratios observed in the experimental data.

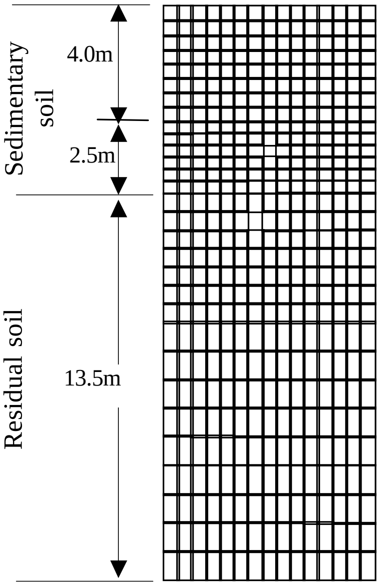


Figure 10. . Soil profile and FEM mesh

The 1D transient flow was simulated with FLOW3D finite element program (Gerscovich, 1994), by imposing a ponding condition on the soil surface. In the beginning of the infiltration process, soil suction was considered constant and equal to 30kPa. This initial condition corresponded to an estimated average value of in-situ soil suction.

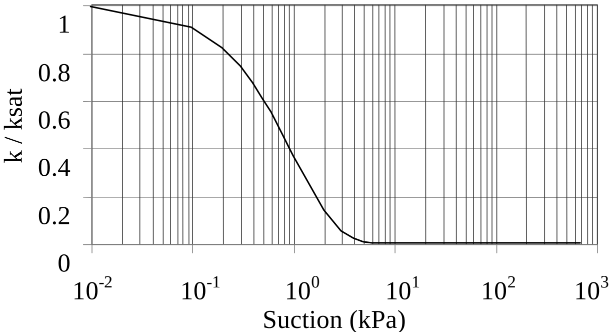


Figure 11. . Relative hydraulic conductivity *vs* suction

Two cases have been analyzed: a homogeneous soil, with hydraulic parameters corresponding to 3m-deep soil sample, and a 3 layers profile, with distinct hydraulic parameters.

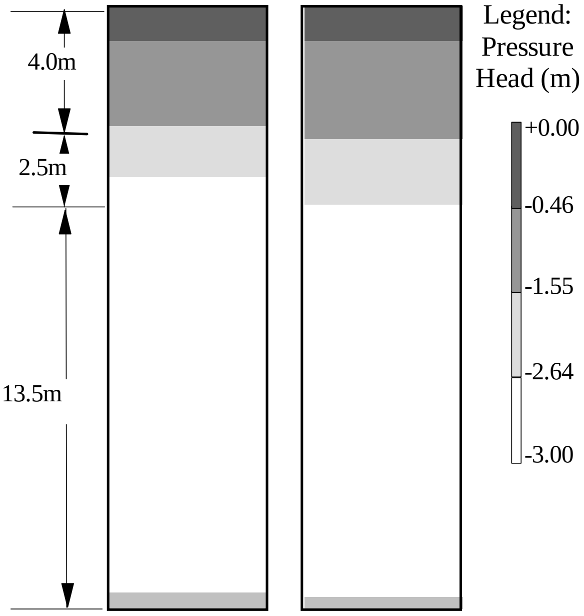
The 2D soil mesh used in this study was composed by 496 nodes and 450 elements, and is also presented in Figure 10.

The distribution of pressure head, after 30 days of flow simulation (Figure 12) indicated no significant influence of the shape of the SWCC on the flow development. In both cases the infiltration front reaches approximately 1m deep. Bellow this point a more intense redistribution of pressure head is observed in the heterogeneous profile. In both cases, no development of positive pore-water pressure occurs.

However, flow simulation depends not only on the shape of SWCC, but also on hydraulic conductivity values. Figure 13 shows soil suction distribution with depth for different relationships between saturated hydraulic conductivity of sedimentary soil ($k_{sat}^{(S)}$) and saturated hydraulic conductivity of residual soil ($k_{sat}^{(R)}$). The analyses indicate that doubling the initial value of $k_{sat}^{(S)}/ k_{sat}^{(R)}$, positive pore water pressures are developed within the first 9m deep. Larger contrasts ($k_{sat}^{(S)} / k_{sat}^{(R)} \geq 16$) result in complete saturation of the sedimentary layer, after 30 days of flow simulation.

The evaluation of a combined influence on flow simulation of both saturated hydraulic conductivity contrasts and soil-water characteristic curves is presented in Figure 14. As previously shown, small differences are observed, when saturated hydraulic conductivity contrast are small. On the other hand, a significant pore-water pressure deviation is verified when larger k_{sat} contrasts are considered. Complete saturation of the whole profile is observed in the homogeneous profile with a relatively high k_{sat}, after 30 days of flow simulation. In the heterogeneous profile, the flow velocities in the underlying soil were controlled by a lower k_{sat} value, which caused a considerable delay in the wetting front.

The small changes on soil suction at the bottom of soil profile are attributed to local pressure head redistribution, due to impervious boundary condition.



(a) homogeneous (b) 3 layers

Figure 12. 1D Flow simulation

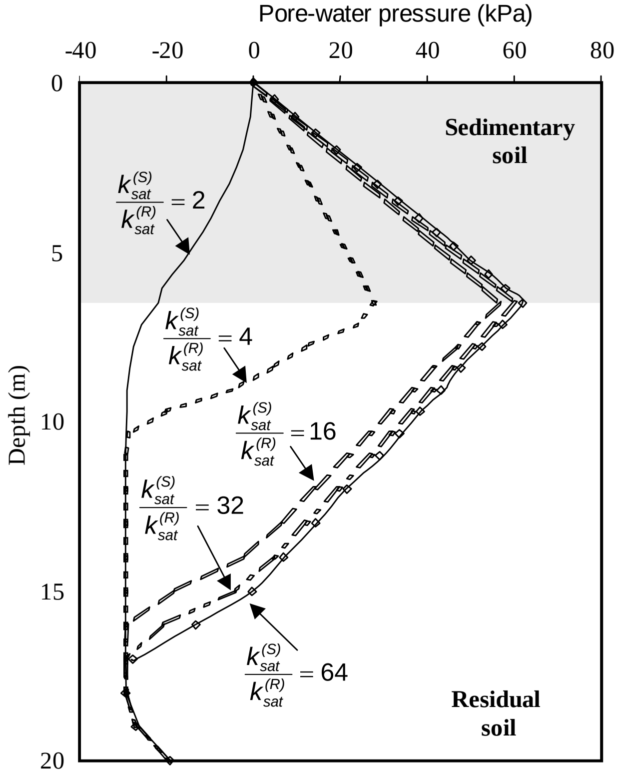


Figure 13. Soil suction profile for different saturated hydraulic conductivity relationships between sedimentary ($k_{sat}^{(S)}$) and residual ($k_{sat}^{(R)}$) soils

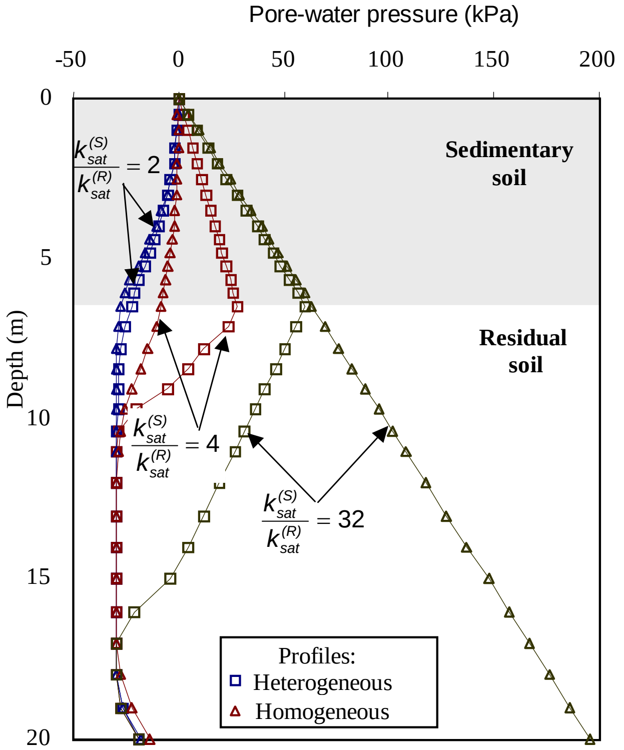


Figure 14. Soil suction profile for different profiles and $k_{sat}^{(S)}$ / $k_{sat}^{(R)}$ relationships.

6. Conclusion

This paper presented an optimization process for evaluating SWCC parameter equations, based on Genetic Algorithms.

An Excel spreadsheet, developed for computing SWCC equations parameters proposed by Gardner (1958) and Fredlund & Xing (1994), was coupled to Evolver commercial program.

Three experimental data from a test site in São Carlos, São Paulo, were analyzed.

Coupling GA optimization method to SWCC parameters spreadsheet revealed to be an easy, quick and effective tool for predicting SWCC fitting parameters. For the present analysis convergence was achieved in less than 1 min. During this period a considerable number of trials were performed. It is worthwhile to note that during the process not a single set of equation parameters was repeated, since all trial results were used to guide the optimum solution

Due to its coupling feature to Excel spreadsheets, GA optimization process became a feasible alternative to be used for solving a variety of problems, associated to a minimum or maximum value in a search space. Moreover, convergence to the best solution is always achieved even in search spaces with local optimum points.

The available experimental data indicated a reasonable variability of the SWCC with depth. Flow simulation through a homogeneous soil profile compared to a heterogeneous material did not produce strong differences on pore pressure head distributions, for small saturated hydraulic conductivity contrasts. Larger contrasts of k_{sat} resulted in strong deviation between soil suction distributions in homogeneous and heterogeneous profiles, after 30 days of flow simulations.

Judgment must be therefore exercised while trying to model the flow behavior using a single set of hydraulic parameters for a given soil.

Acknowledgements

The authors gratefully thank CNPq for sponsoring this research project.

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Table 1. Soil-water characteristic curve equations estimated from the shape of pore size distribution

References	Equation	Parameters
Gardner (1858)	$\Theta = \frac{1}{1+q\psi^{\eta}}$	Θ = normalized volumetric water content = $(\theta\text{-}\theta_r)/(\theta_s\text{-}\theta_r)$ η e α = fitting parameters
Brooks & Corey (1964)	$\Theta = \left(\frac{\psi_b}{\psi}\right)^{\lambda}$	Θ = normalized volumetric water content: λ = fitting parameter
Visser (1966)	$\psi = a(\theta_s - \theta)^b / \theta^c$	a b e c = fitting parameters
Farrel & Larson (1972)	$\psi = \psi_b e^{\alpha(1-\Theta)}$	α= fitting parameter
Roger & Hornberger (1978)	$\psi = a(S_s - b)(S_s - 1)$	$S_s = \theta/\theta_s$: a e b = fitting parameters
Van Genuchten (1980)	$\Theta = \left[\frac{1}{1+(\alpha\psi)^n}\right]^m$	Θ = normalized volumetric water content: α, m e n = fitting parameters
William et al (1983) Saxton et al (1986)	$\ln\psi = a + b\ln\theta$	a e b = fitting parameters
Haverkamp & Parlange (1986)	Wetting path: $\theta = \frac{n}{1+\lambda}\left(\frac{\psi_b}{\psi}\right)^{\lambda} \ldots \psi > \psi_b$ $\theta = n\left[1-\left(\frac{\lambda}{1+\lambda}\right)\left(\frac{\psi}{\psi_b}\right)\right] \ldots \psi_b^* \le \psi \le \psi_b$ $\theta = \theta_s \ldots \psi \le \psi_b^*$ Drying path: $\theta = n\left(\frac{\psi_b}{\psi}\right)^{\lambda}\left[1-\frac{\psi_b}{\psi}\left(1-\frac{\theta_s}{n}\right)\right] \ldots \psi > \psi_b$ $\theta = \theta_s \ldots \psi \le \psi_b$	λ = fitting parameter θs related to absorption curve
McKee e Bumb (1987)	$\Theta = \frac{1}{1+e^{(\psi-a)/b}}$	Θ = normalized volumetric water content a e b = fitting parameters
Fredlund e Xing (1994)	$\theta = C_\psi \frac{\theta_s}{\left[ln[e+(\psi/a)^n]\right]^m}$ $C_\psi = 1 - \frac{ln(1+\psi/\psi_r)}{ln(1+\psi_o/\psi_r)}$	a, m e n = fitting parameters e = 2,718 ψ_o = dry soil matric suction (10^6kPa) $\psi_o = 10^6$kPa a = matric suction related to ψ_i θ_i = volumetric water content related to ψ_i $m = 3{,}67\ln\left(\frac{\theta_s}{\theta_i}\right)$; $n = \frac{1{,}31^{m+1}}{m\theta_s}3{,}72s\psi_i$
Aubertin et al (1998)	$S_r = S_c + S_a(1-S_c)$ $S_c = 1-\left[\left(\frac{h_{co}}{\psi}\right)^2+1\right]e^{\left[-m\left(h_{co}/\psi\right)^2\right]}$ $S_a = C_\psi \frac{a}{e^{1/3}\psi^{1/6}}\psi_{90}^{2/3}$ $C_\psi = 1-\frac{ln(1+\psi/\psi_r)}{ln(1+\psi_o/\psi_r)}$	a, h_{co} e m = fitting parameters h_{co}=1 to 2,5ψ_b (cm H_2O) $S_r = \theta/\theta_s$: a ≈ 0,006 $\psi_o = 10^7$ (cm H_2O) $\psi_r = 15x10^3$ (cm H_2O) ⇔ θ_r e = void ratio

Table 2. Soil-water characteristic curve equations estimated from the grain size distribution and physical properties of soils.

References	Equation	Parameters
Gosh (1980)	$\psi = \psi_b \left(\frac{\theta}{\theta_s} \right)^{-\beta}$ $\beta = 2{,}619 \left(\frac{\lambda_2}{\lambda_1} \right)^{0{,}2822} (\lambda_4 + 0{,}7)^{0{,}0625} \lambda_4^{0{,}1250} \left(5{,}91 \frac{\lambda_3}{\lambda_1 + \lambda_3} + 1{,}1 \right)^{0{,}0625}$	λ_1 percentage of sand fraction λ_2 percentage of silt fraction λ_3 percentage of clay fraction $\lambda_4 = 6{,}2 \sqrt{\lambda_2 / \lambda_1} - 5{,}91 \frac{\lambda_3}{\lambda_1 + \lambda_3}$
Rawls & Brakensiek (1989)	$\Theta = \left(\frac{\psi_b}{\psi} \right)^{\lambda}$ $\psi_b = e^{\left[\begin{array}{l} 5{,}3396738 + 0{,}1845038(C) - 2{,}438394546(n) - 0{,}00213853(C^2) - \\ 0{,}04356349(S)(n) - 0{,}61745089(C)(n) + 0{,}00143598(S^2)(n^2) - \\ 0{,}00855375(C^2)(n^2) - 0{,}00001282(S^2)(C) + 0{,}00895359(C)^2(n) - \\ 0{,}00072472(S^2)(n) + 0{,}0000054(C^2)(S) + 0{,}50028060(n^2)(C) \end{array} \right]}$ $\lambda = e^{\left[\begin{array}{l} -0{,}7842831 + 0{,}0177544(S) - 1{,}062498(n) - 0{,}000005304(S^2) - \\ 0{,}00273493(C^2) + 1{,}11134946(n^2) - 0{,}03088295(S)(n) + \\ 0{,}00026587(S^2)(n^2) - 0{,}00610522(C^2)(n^2) - \\ 0{,}00000235(S^2)(C) + 0{,}00798746(C^2)(n) - 0{,}00674491(n^2)(C) \end{array} \right]}$	ψ_b (cm H_2O) C = percentage of clay fraction (5%<C< 60%) S = percentage of sand fraction (5%<S< 70%) $\theta_r = -0{,}018242 + 0{,}00087269(S) + 0{,}00513488(C) + 0{,}02939286(n) - 0{,}00015395(C^2) - 0{,}0010827(S)(n) - 0{,}00018233(C^2)(n^2) + 0{,}00030703(C^2)(n) - 0{,}0023584(n^2)(C)$

Table 3. **Soil Characterization**

Depth (m)	3	5	8
Soil type	Sedimentary		Residual
ω_{nat} (%)	14.20	16.40	16.70
γ_t (kN/m³)	15.60	17.40	19.20
γ_s (kN/m³)	27.10	27.50	27.10
e	0.98	0.84	0.65
S (%)	39.87	54.73	71.30
θ_{nat} (%)	19.40	24.52	27.48
ψ_{nat} (kPa)	25.0	30.0	8.0
θ_s (%)	47.0	43.0	37.0
θ_r (%)	15.0	18.0	15.0
Clay (%)	27.30	27.40	17.40
Silt (%)	11.90	5.90	13.70
Sand (%)	60.80	66.70	68.90

Notes: ω_{nat} ; θ_{nat} = in situ gravimetric and volumetric water content; ψ_{nat} = in situ soil suction; γ_t = in situ density; γ_s = dry density; e= void ratio, S = in situ degree of saturation, θ_s = saturated volumetric water content, θ_r = residual volumetric water content

Table 4. Variable constraints

Equation	Parameter interval
Gardner	$0 < \alpha \leq 10$ $0 < \eta \leq 3$ $(0 < \theta^*_r \leq \theta_s)$
Fredlund & Xing	$0 < n \leq 10$ $0 < m \leq 10$ $0 < a \leq 10$ KPa.

Table 5. **Fitted Parameters and computed errors.**

Eq.	Search type	Parameter	Depth (m)		
			3	5	8
Gardner	Manual	α	0.54	0.15	0.08
		η	0.76	0.76	0.75
		Error	0.039	0.033	0.036
	Numerical (GA)	α	0.47	0.19	0.10
		η	0.873	0.95	0.91
		θr^* (%)	15.9	20	17
		Error	0.021	0.026	0.032
	Convergence Time (min:s)		00:56	01:00	01:26
Fredlund & Xing	Manual	n	1.30	1.30	1.20
		m	0.53	0.41	0.43
		a (kPa)	0.50	1.30	2.30
		Error	0.036	0.032	0.033
	Numerical (GA)	n	1.32	1.30	1.18
		m	0.53	0.41	0.43
		a (kPa)	0.529	1.25	2.21
		Error	0.035	0.031	0.032
	Convergence Time (min:s)		01:00	00:35	00:58

Appendix 1. Notation

GA	genetic algorithm
ε	computed error
θ_{nat}	in situ volumetric water content
θ_i	predicted volumetric water content
$\hat{\theta}_i$	measured volumetric water content
θ_r	residual volumetric water content
θ_r^*	residual volumetric water content, computed by Evolver
θ_{sat}	saturated volumetric water content
ω_{nat}	in situ gravimetric water content
ψ	matric suction
ψ_r	matric suction corresponding to θ_r
ψ_o	matric suction corresponding to dry soil
Θ	normalized volumetric water content
α , η	Gardner’s equation parameters
a, m, n	Fredlund & Xing’s equation parameters
k	hydraulic conductivity
k_{sat}	saturated hydraulic conductivity
α_1	hydraulic conductivity equation parameter
k/k_{sat}	relative hydraulic conductivity
γ_t	in situ density
γ_s	dry density
n	porosity
e	void ratio
(R)	Residual soil
S	in situ degree of saturation
(S)	Sedimentary soil