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# Evaluation of the soil-water characteristic curve equations for soils from Brazil

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ABSTRACT: The relationship between the soil-water content and matric suction is conventionally referred to as the soil-water characteristic curve, (SWCC). The SWCC is a useful tool in the prediction of the engineering behavior of unsaturated soils. Several equations are available in the literature to mathematically represent the experimental behavior of the SWCC. Some SWCC equations are based on the assumption that the shape of the curve is dependent upon pore size distribution. Other equations assume that the SWCC can be estimated from the grain size distribution and the physical properties of soils. This paper discusses the applicability of four different SWCC equations for eleven soils from Brazil. The experimental data of the eleven soils include residual, colluvial and sedimentary soils from different sites. The studies show that the Gardner's equation is simple and most convenient as it defines the SWCC data of different soils with the smallest number of required curve parameters. In this paper, several parameters that influence the SWCC behavior are also discussed.

## 1 INTRODUCTION

Several numerical models for simulating flow behavior through unsaturated porous media have been proposed in the last two decades, as direct measurements are time consuming and expensive, both with respect to field and laboratory studies (Bear & Verruijt 1987, van Genuchten 1980 Fredlund & Rahardjo 1993). Several numerical models/empirical functions are available in the literature for predicting the flow behavior in unsaturated soils. In all these functions, it is a common engineering practice to use the soil-water characteristic curve (SWCC) as a tool. The SWCC data is used in the form of a mathematical equation in the numerical models/empirical functions.

This paper evaluates the suitability of using four different SWCC equations for defining the relationship between water content and soil suction of eleven Brazilian soils. Also, various parameters that influence the SWCC behavior are briefly discussed.

## 2 SOIL-WATER CHARACTERISTIC CURVE

The soil-water characteristic curve (SWCC) is the relationship between the soil suction and the volume of water in the soil pores. The water content can be established in volumetric ($\theta$) or gravimetric ($\omega$) terms or, alternatively expressed by the degree of saturation ($S$). The gravimetric water content is most commonly used in geotechnical engineering practice. In soil science, the volumetric water content is usually adopted and the SWCC is referred to as the soil-water retention curve.

Soil matric suction ($\psi$) is defined as the difference between pore-air pressure ($u_a$) and pore-water pressure ($u_w$). Total suction ($\psi_t$) is equal to the sum of osmotic suction and matric suction. For practical engineering applications it can be assumed that the total suction is equal to osmotic suction for high values of soil suction (above 1500kPa).

The shape of the SWCC is dependent on soil mineralogy and grain size distribution, which is intrinsically related to pore-size distribution. Sandy soils show a sharp loss of water content for relatively low soil suction values. Clayey soils usually show a more gentle SWCC behavior (Fig. 1a). Silty soils exhibit an intermediate behavior. Uniformly graded soils have SWCC similar to sandy soils, while well-graded soils can be compared to clayey soils.

Stress state, compaction energy and soil structure are other parameters that influence the shape of the SWCC for fine-grained soils (Vanapalli et al. 1999). The compaction of an originally undisturbed soil causes a reduction of the volume of the largest soil pores (i.e., macroscopic pores) and has little or no effect on the small size pores (i.e., microscopic pores). Therefore, there is an increase in the percentage of soil pores with intermediate volume, resulting in a flatter shape of the SWCC (Fig. 1b), for low levels of soil suction (Gerscovich 1984).

The usual S-shape of the SWCC may be defined by four parameters: saturated volumetric water content ($\theta_s$); residual volumetric water content ($\theta_r$); air-entry value or bubbling pressure ($\psi_b$) and volumetric water retention capacity ($\Delta\psi/\Delta\theta$), shown in Figure 2.

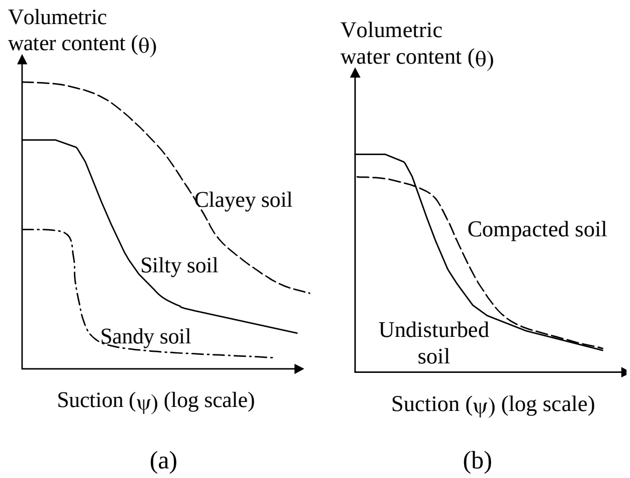


Figure 1. Typical soil-water characteristic curves: (a) Influence of grain size distribution; (b) Influence of soil structure.

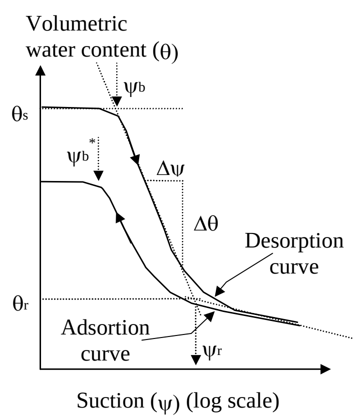


Figure 2. Soil-water characteristic curve parameters.

The volumetric water content ($\theta$), defined by the ratio between pore water volume and total volume, is equivalent to porosity ($n$) at full saturation.

In a gradually wetting SWCC of an initially dry soil (adsorption curve), the water content values are lower in comparison to a drying SWCC at any value of suction (desorption curve). As a result, full saturation condition of 100% is rarely achieved during wetting processes. This hysteretic pattern is mainly attributed to geometric non-uniformities of the interconnected pores and/or to entrapped air. Soil structure changes due to swelling or shrinking phenomena are also partially responsible for the hysterisis (Hillel 1971). It has been experimentally observed that the difference in $\theta$ -values increases with the percentage of coarse particles (Smith & Browning 1942; Wilson et al. 1981).

The bubbling pressure ($\psi_b$) defines the soil suction at which water in the largest pores starts to drain. The $\psi_b$ value is relatively small and depends on the pore-size of the soil. It is expected a range of $\psi_b$ = 0.2 to 7.5kPa for coarse to fine sands, $\psi_b$ = 7 to 25kPa for silty soils, and a $\psi_b$ > 25kPa for clays (Aubertin et al. 1998). The bubbling pressure can be graphically estimated, as shown in Figure 2. Aubertin et al. (1998) proposed that $\psi_b$ may be considered as the matric suction corresponding to $\theta = 0.9\ \theta_s$. At this volumetric water content, these authors suggest that continuous channels are created within the soil. This approach results in $\psi_b$ values 25% higher than those obtained by the graphical method in Figure 2.

The residual volumetric water content ($\theta_r$) is a lower limit, beyond which an increase in matric suction does not significantly reduce the soil water content. Value of $\theta_r$ can also be graphically obtained, as shown in Figure 2.

## 3 SOIL-WATER CHARACTERISTIC CURVE EQUATIONS

A number of equations are available in the literature to mathematically represent the SWCC data. These equations in turn are used in empirical functions to model the flow behavior in unsaturated soils. Most equations are based on the assumption that the shape of this curve is dependent upon pore size distribution (Gardner 1958; Brooks & Corey 1964; Farrel & Larson; 1972; van Genuchten 1980; William et al.1983; Saxton 1986; Haverkamp & Parlange 1986; McKee & Bumb 1987; Fredlund & Xing 1994). This assumption implicitly considers a spherical shape for the water-air meniscus in the pores. It is therefore assumed a cylinder shape for the interconnected channels within the soil. The parameters for these equations are calibrated by linear regression of experimental data.

Other procedures assume that the SWCC can be directly estimated from the grain size distribution and physical properties of soils (Ghosh 1980; Rawls & Brakensiek 1989). These simple procedures are convenient in engineering practice, because grain size distributions can be determined in all conventional soil laboratories. However, these procedures disregard stress state, soil structure, compaction water content, compaction energy and mineralogy that play a major role in defining the shape of the SWCC and influence the flow behavior in unsaturated soils.

Gerscovich (2001) studied the suitability of 14 different SWCC equations for fitting the experimental data of the SWCC for soils from Brazil. The SWCC equations provided by Gardner (1958), van Genuchten (1980), Haverkamp & Parlange (1986) and Fredlund & Xing (1994) provided better fits to the SWCC for two residual Brazilian soils.

The first equation was proposed by Gardner (1958) and is defined as:

$$\Theta = \frac{1}{1+\alpha\psi^{n}} \qquad (1)$$

where $\alpha$ and $n$ are equation parameters and $\Theta$ is the normalized volumetric water content = $(\theta-\theta_r)/(\theta_s-\theta_r)$.

The proposition by van Genuchten (1980) is similar to the previous one and is written as:

$$\Theta = \left[\frac{1}{1+\alpha\psi^{n}}\right]^{m} \qquad (2)$$

This equation includes an additional curve parameter ($m$), which gives more flexibility for curve fitting. The author suggests that $m$=1-1/$n$.

Haverkamp & Parlange (1986) proposed equations for non-shrinking non-organic sandy soils. The authors considered a shape similarity between the SWCC and the cumulative particle-size distribution function. In the authors' approach, a hysteretic model is associated to the Brooks-Corey (1964) equation, assuming a null value for $\theta_r$. The Haverkamp-Parlange equations are:

i) Adsorption curve:

$$\theta = \frac{n}{1+\lambda}\left(\frac{\psi_b}{\psi}\right)^{\lambda} \ldots \psi > \psi_b$$
$$\theta = n\left[1-\left(\frac{\lambda}{1+\lambda}\right)\left(\frac{\psi}{\psi_b}\right)\right] \ldots \psi_b^{*} \le \psi \le \psi_b \qquad (3)$$
$$\theta = \theta_s \ldots \psi \le \psi_b^{*}$$

ii) Desorption curve:

$$\theta = n\left(\frac{\psi_b}{\psi}\right)^{\lambda}\left[1-\frac{\psi_b}{\psi}\left(1-\frac{\theta_s}{n}\right)\right] \ldots \psi > \psi_b \qquad (4)$$
$$\theta = \theta_s \ldots \psi \le \psi_b$$

where $n$ = porosity, $\psi_b$ and $\psi_b^{*}$ = air-entry pressures for the drying and wetting curves, respectively, $\theta_s$ = saturated volumetric water content and $\lambda$ is an equation parameter related to pore size distribution.

Fredlund & Xing (1994) provided a theoretical basis for mathematically representing the SWCC using the pore-size distribution curve. The proposed equation primarily considers the desorption curve and is written as:

$$\theta = \left[1-\frac{\ln(1+\psi/\psi_r)}{\ln(1+\psi_o/\psi_r)}\right]\frac{\theta_s}{\left[\ln[e+(\psi/a)^{n}]\right]^{m}} \qquad (5)$$

where $a$, $m$ and $n$ are equation parameters, $e$ is a constant equal to 2.718, $\psi_r$ is matric suction corresponding to the residual volumetric water content ($\theta_r$) and $\psi_o$ is the matric suction corresponding to dry soil ($10^6$kPa). The authors suggest a numerical, procedure for estimating fitting parameters a, m and n from the experimental SWCC data. The first term in Equation 5 is a correction factor for high-suction range.

## 4 EXPERIMENTAL RESULTS

Several researches carried out laboratory tests to determine the SWCC for Brazilian soils (Table 1). Most tests made use of tension plate devices for low soil suction range ($\psi$ < 100kPa) and pressure cells for higher matric suction values. In these tests, suction values are imposed and the corresponding equilibrium soil water contents were measured.

Table 1 presents all test sites considered in this paper and Table 2 summarizes soil characterization results from all sites. The residual soils are predominantly sandy, while colluvial soils have a higher percentage of clay fraction due to weathering processes. The sedimentary soils at São Carlos site were obtained from two different depths (3m-Test #9 and 5m-Test #10). At this same site, a residual soil was also investigated (8m-Test #5).

Table 1. Test sites description.

| Test # | Soil | Site | Reference |
|---|---|---|---|
| 1 | residual | Vista Chinesa, RJ | Delgado, 1993 |
| 2 | | Querosene, RJ | Souza, 1995 |
| 3 | | Lagoa, RJ | Fonseca, 1991 |
| 4 | | Salvador, BA | Machado & Lima Jr, 2001 |
| 5 | | São Carlos, SP | Machado & Vilar, 1998 |
| 6 | | Porto Alegre, RS | Oliveira et al, 2001 |
| 7 | colluvial | Vista Chinesa, RJ | Delgado, 1993 |
| 8 | | Querosene, RJ | Souza, 1995 |
| 9 | sedimentary | São Carlos, SP | Machado & Vilar, 1998 |
| 10 | | São Carlos, SP | |
| 11 | | Brasilia, DF | Peixoto et al, 2001 |

## 5 SOIL-WATER CHARACTERISTIC CURVE PREDICTION

A spreadsheet for curve fitting was developed for computing all equation parameters for the 4 propositions (Gerscovich 2001). The curve shape parameters ($\theta_s$, $\theta_r$ and $\psi_b$) were directly inferred from experimental data and are summarized in Table 3. The residual soil suction ($\psi_r$) was considered constant and equal to $10^4$kPa. This is a reasonable assumption for the tested soils (Gerscovich 2001).

Table 3 also shows the total number of experimental data ($N$) and soil porosity ($n$). The ratio between porosity and volumetric water content $\theta_s$ is equal to 1.0 at full saturation condition. Tests following a wetting (adsorption) path usually indicate an $\theta_s$ value smaller than porosity, for $\psi$ = 0. In this paper, $\theta_s$ was considered to be equal to 0.90*n*, due to the lack of data at low suctions for adsorption test paths. For tests carried out with both adsorption and desorption paths, $\theta_s$ was estimated as 0.95*n*. These assumptions are close to the suggestions provided by Aubertin et al. (1998).

Table 2. Soil-characterization from different tests.

| Test # | $\omega_{nat}^{*}$ (%) | $\gamma_t^{**}$ (kN/m$^3$) | e$^{***}$ | Clay (%) | Silt (%) | Sand (%) |
|---|---|---|---|---|---|---|
| 1 | 18.9 | 14.54 | 1.24 | 8.11 | 11.3 | 65.6 |
| 2 | 5.3 | 14.75 | 0.94 | 10.0 | 15.0 | 75.0 |
| 3 | 28.9 | 14.00 | 1.19 | 9.5 | 27.5 | 63.0 |
| 4 | 28.9 | 16.02 | 1.10 | 22.0 | 17.0 | 61.0 |
| 5 | 16.7 | 19.20 | 0.65 | 17.4 | 13.7 | 68.9 |
| 6 | 17.2 | 17.00 | 0.84 | 14.0 | 19.0 | 64.0 |
| 7 | 19.2 | 14.90 | 1.18 | 41.2 | 4.5 | 50.5 |
| 8 | 28.7 | 14.51 | 1.43 | 59.0 | 8.0 | 33.0 |
| 9 | 14.2 | 15.60 | 0.98 | 27.3 | 11.9 | 60.8 |
| 10 | 16.4 | 17.40 | 0.84 | 27.4 | 5.9 | 66.7 |
| 11 | 17.5 | 12.94 | 1.47 | 14.3 | 44.6 | 41.1 |

* $\omega_{nat}$ = in situ gravimetric water content; ** $\gamma_t$ = in situ density; ***e= void ratio.

Table 3. Soil-water characteristic curve parameters ($\theta_s$, $\theta_r$, $\psi_b$), soil porosity (*n*) and number of experimental data (*N*)

| Test # | *N* | *n* (%) | $\theta_s$ (%) | $\theta_r$ (%) | $\psi_b$ (kPa) |
|---|---|---|---|---|---|
| 1 | 6 | 55.0 | 52.0 | 10.0 | 1.0 |
| 2 | 7 | 48.0 | 44.0 | 7.0 | 1.0 |
| 3 | 5 | 54.0 | 49.0 | 7.0 | 1.0 |
| 4 | 5 | 52.0 | 52.0 | 20.0 | 1.0 |
| 5 | 16 | 39.0 | 39.0 | 15.0 | 1.0 |
| 6 | 9 | 46.0 | 41.0 | 15.0 | 1.0 |
| 7 | 8 | 54.0 | 54.0 | 15.0 | 1.0 |
| 8 | 8 | 59.0 | 59.0 | 20.0 | 1.0 |
| 9 | 16 | 50.0 | 47.0 | 15.0 | 1.0 |
| 10 | 16 | 46.0 | 43.0 | 18.0 | 1.0 |
| 11 | 7 | 59.5 | 45.0* | 18.0 | 1.0 |

* Experimental result

Air-entry values ($\psi_b$) were assumed to be equal to 1kPa for all tests, despite different percentages of granular and fine materials. It is very difficult to define $\psi_b$ based on experimental data, because of limitations of laboratory techniques for applying suction values lower than 1kPa. Nevertheless, this parameter was observed to have no influence on curve fitting, except for the Haverkamp-Parlange equation.

Usual practice for assessing the quality of curve prediction is based on the correlation coefficient ($r^2$) of a plot between predicted and measured values. Values of $r^2$ close to 1.0 would indicate a good prediction only when a linear relationship is obtained, together with an angular coefficient equal to 1.0 and a null intercept.

An alternative error criterion ($\varepsilon$) is herein proposed to assess the quality of curve fitting the SWCC data:

$$\varepsilon = \frac{1}{N}\sum_{i=1}^{N}\left(\theta_i - \hat{\theta}_i\right)^2 \quad (6)$$

where $\theta_i$ and $\hat{\theta}_i$ = predicted and measured volumetric water contents, respectively, and *N* = number of experimental data.

Previous studies have shown that the SWCC equations with computed errors $\varepsilon < 4.0$ provide a good fit with the experimental data (Gerscovich, 2001).

Table 4 shows fitted parameters and computed errors for all the SWCC equations. Except for the Haverkamp-Parlange proposition, all equations predicted experimental data fairly well, with $\varepsilon$ well below 4.0.

Figure 3 shows computed errors ($\varepsilon$) for different test sites and equations. The results show that the equations by Gardner (1958), van Genuchten (1980) and Fredlund & Xing (1994) provide good fits of the SWCC for the tested soils from Brazil. It is worthwhile to notice the similar trend in $\varepsilon$ variation for all sites, which suggest that largest errors might be associated to the poorest quality of experimental data. It must be noted that the quality of curve predictions is not only influenced by the $\theta$ - $\psi$ experimental data, but also by the values of soil porosity and saturated volumetric water content values.

Haverkamp & Parlange (1986) proposition has the advantage of providing different equations for adsorption and desorption curves. However, the difference between both curves has been experimentally observed to be negligible for Brazilian residual soils (Fonseca 1991; Delgado 1993; Souza 1995; Oliveira et al. 2001). The computed errors presented in Table 4 correspond to the minimum values obtained by drying and wetting equations. The relatively high errors were attributed to the uncertainties in air-entry values ($\psi_b$) and to the different (wetting and drying) paths used for obtaining $\theta$ - $\psi$ experimental data. The difference between the air-entry pressures, corresponding to the drying ($\psi_b$) and wetting ($\psi_b^*$) paths, were disregarded.

A comparison among the three equations, which gave $\varepsilon < 4.0$, Gardner (1958) equation appears to be the most convenient one. This is due to the reason it is simple and requires the smallest number of fitting parameters.

The van Genuchten (1980) equation is similar to Gardner (1958), but includes an additional parameter (*m*). When *m*=1.0, both propositions become identical. The results, shown in Table 4, indicate that no fixed relationship between parameters m and n provide the best curve fitting. Similar conclusions have been drawn by Fredlund & Xing (1994).

The prediction of soil-water characteristic curve does not seem to be sensitive to the total number of experimental data, provided there are sufficient data over a reasonable range of soil suction. Leong and Rahardjo (1998) observed significant deviation between measured and predicted values, if data points with $\theta < \theta_r$ are not included; i.e., if the region of high matric suction is not sufficiently covered.

Table 4. Fitted Parameters and computed errors (Eq. 6)

(a) Gardner Equation

| Soil | Site | α | n | Error |
|---|---|---|---|---|
| Residual | Vista Chinesa | 0.22 | 0.58 | 0.63 |
| | Querosene | 0.17 | 0.68 | 0.47 |
| | Lagoa | 0.04 | 1.65 | 0.78 |
| | Salvador | 0.01 | 0.29 | 0.37 |
| | São Carlos | 0.10 | 0.70 | 0.69 |
| | Porto Alegre | 0.12 | 0.50 | 0.91 |
| Colluvial | Vista Chinesa | 0.82 | 0.72 | 0.71 |
| | Querosene | 0.40 | 0.60 | 0.66 |
| Sedimentary | São Carlos | 0.54 | 0.76 | 0.80 |
| | São Carlos | 0.15 | 0.76 | 0.86 |
| | Brasilia | 0.03 | 0.39 | 0.58 |

(b) van Genuchten Equation

| Soil | Site | α | n | m | Error |
|---|---|---|---|---|---|
| Residual | Vista Chinesa | 0.86 | 1.50 | 0.29 | 0.72 |
| | Querosene | 0.68 | 1.45 | 0.33 | 0.90 |
| | Lagoa | 0.06 | 1.80 | 0.70 | 1.09 |
| | Salvador | 0.46 | 1.20 | 0.13 | 0.79 |
| | São Carlos | 0.53 | 1.36 | 0.32 | 0.80 |
| | Porto Alegre | 1.00 | 1.20 | 0.25 | 1.16 |
| Colluvial | Vista Chinesa | 1.80 | 1.60 | 0.38 | 0.54 |
| | Querosene | 1.50 | 1.70 | 0.28 | 0.57 |
| Sedimentary | São Carlos | 1.30 | 1.40 | 0.40 | 0.46 |
| | São Carlos | 0.60 | 1.60 | 0.32 | 0.77 |
| | Brasilia | 0.45 | 0.60 | 0.38 | 0.32 |

(c) Haverkamp-Parlange Equation

| Soil | Site | λ | Error |
|---|---|---|---|
| Residual | Vista Chinesa | 0.23 | 4.00 |
| | Querosene | 0.22 | 1.47 |
| | Lagoa | 0.20 | 55.0 |
| | Salvador | 0.06 | 0.54 |
| | São Carlos | 0.16 | 2.20 |
| | Porto Alegre | 0.13 | 0.99 |
| Colluvial | Vista Chinesa | 0.24 | 13.70 |
| | Querosene | 0.22 | 11.30 |
| Sedimentary | São Carlos | 0.22 | 8.03 |
| | São Carlos | 0.14 | 1.81 |
| | Brasilia | 0.14 | 7.19 |

(d) Fredlund-Xing Equation

| Soil | Site | a(kPa) | n | m | Error |
|---|---|---|---|---|---|
| Residual | Vista Chinesa | 0.70 | 1.00 | 0.72 | 0.86 |
| | Querosene | 1.50 | 1.18 | 0.80 | 1.01 |
| | Lagoa | 12.00 | 2.20 | 0.95 | 1.58 |
| | Salvador | 3.00 | 0.60 | 0.36 | 0.65 |
| | São Carlos | 1.50 | 1.20 | 0.43 | 0.76 |
| | Porto Alegre | 1.20 | 1.00 | 0.47 | 1.27 |
| Colluvial | Vista Chinesa | 0.80 | 3.90 | 0.39 | 0.62 |
| | Querosene | 1.00 | 2.10 | 0.40 | 0.87 |
| Sedimentary | São Carlos | 0.50 | 1.30 | 0.53 | 0.68 |
| | São Carlos | 1.30 | 1.30 | 0.41 | 0.77 |
| | Brasilia | 1.10 | 0.50 | 0.47 | 0.51 |

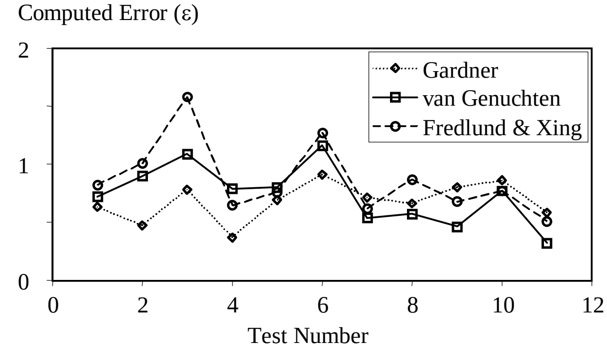


Figure 3. Computed errors from three selected equations.

Figure 4 shows Gardner's predictions and corresponding experimental results for two different depths at São Carlos site. In spite of referring to the same material, the curves are distinct and, therefore, the equation parameters are independent. This result denotes the variability of the SWCC and, consequently, the difficulty in defining a single curve for describing an entire soil layer. This behavior plays an important role for proper simulation of transient flow through unsaturated soil layers. These simulations require the knowledge of volumetric water retention capacity ($\Delta\psi/\Delta\theta$), which relates the variation of soil water content to the corresponding deviation of soil suction. This parameter is strongly influenced by the shape of the SWCC. Figure 5 compares water retention capacities, computed from Gardner's equation, for São Carlos site. It is observed that the differences are about 50% at high suctions and can achieve 200% at low suction values.

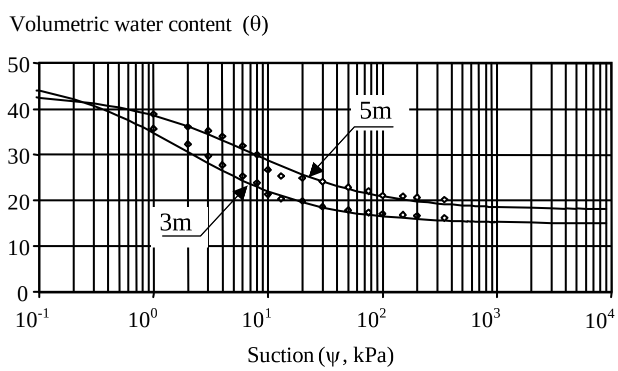


Figure 4. Soil-water characteristic curves - São Carlos sedimentary soil at different depths (3m-Test #9 and 5m-Test #10)

## 6 CONCLUSIONS

The equations proposed by Gardner (1958), van Genuchten (1980) and Fredlund & Xing (1994) may be used to provide a good estimate of the SWCC for the soils from Brazil. Gardner's equation requires the smallest number of constants and therefore appears to be the most convenient one.

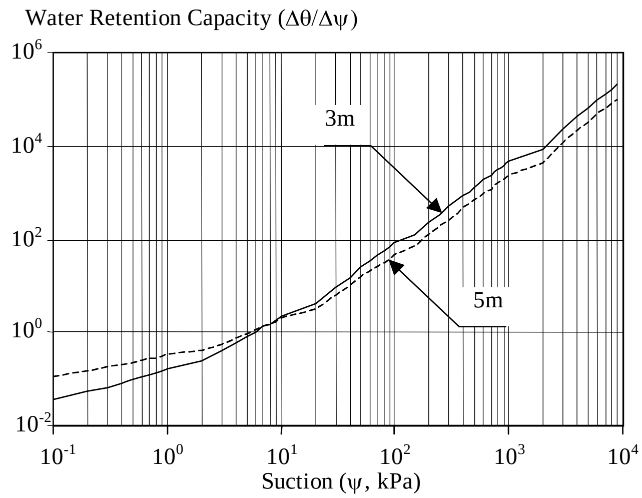


Figure 5. Water retention capacity - São Carlos sedimentary soil at different depths (3m-Test #9 and 5m-Test #10).

All the three equations require definition of curve parameters, the most important being the saturated ($\theta_s$) and residual ($\theta_r$) volumetric water contents. The bubbling pressure ($\psi_b$) appears to have no influence on the results. The accuracy of curve prediction is also strongly dependent on the water content path.

The SWCC behavior varies significantly within a soil layer, possibly due to differences in soil mineralogy, stress state, pore-size distribution and/or soil structure. Judgment must be therefore exercised while trying to model the flow behavior using a single SWCC for a given soil.

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